Competitive Bidding Strategies for Online Linear Optimization with Inventory Management Constraints

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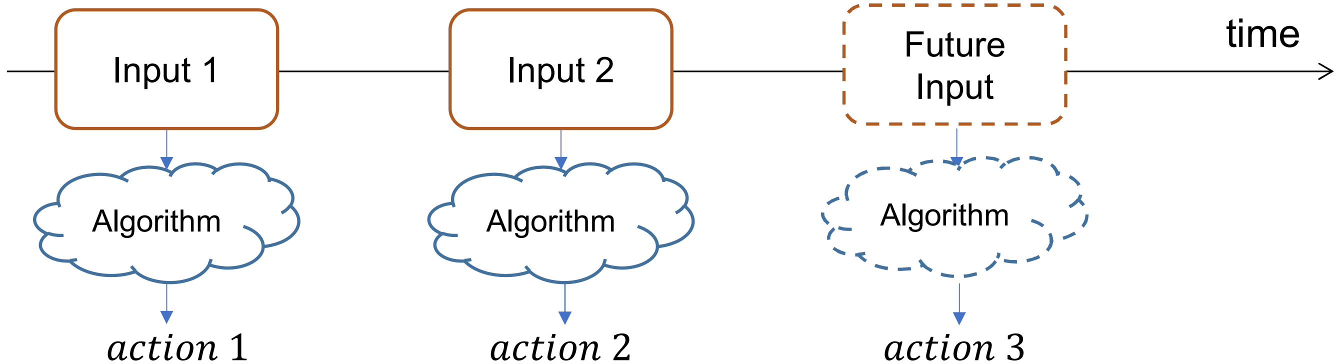
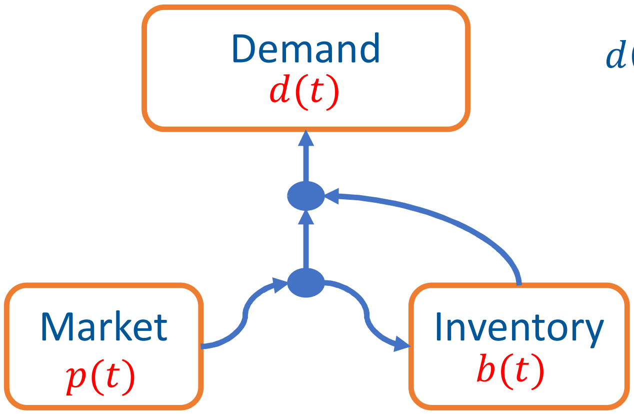
Online Algorithms

- Inputs: only the history up to current point known
  - Make no assumptions about future input

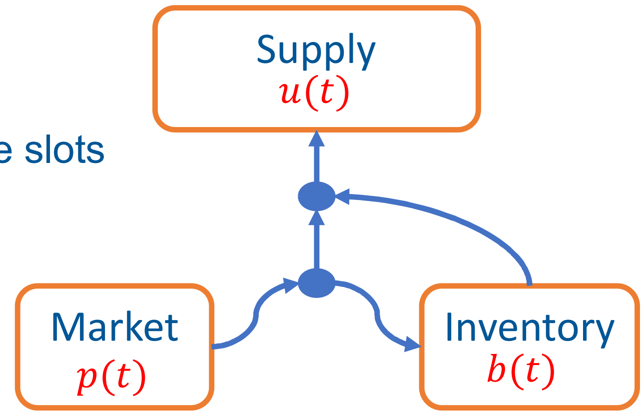


- Competitive Analysis: compare against optimal offline algorithm
  - Competitive ratio: $CR(Alg^{on}) = \max_{\sigma \in instances} \frac{Cost(Alg^{on}, \sigma)}{Cost(Alg^{off}, \sigma)}$

Online Linear Optimization with Inventory Management



$d(t), u(t)$ and $p(t)$
- -- known for current slot
- -- unknown for the future slots



$$
\begin{aligned}
\min \quad & \sum_{t \in \mathcal{T}} p(t)x(t) && \rightarrow \text{Minimize cost} \\
\text{s.t. :} \quad & \forall t \in \mathcal{T}: \\
& x(t) \geq d(t) - b(t-1), && \rightarrow \text{Cover the demand} \\
& b(t) = b(t-1) + x(t) - d(t), && \rightarrow \text{Inventory Management} \\
\text{vars. :} \quad & x(t) \geq 0, t \in \mathcal{T}.
\end{aligned}
$$

$$
\begin{aligned}
\max \quad & \sum_{t \in \mathcal{T}} p(t)y(t) && \rightarrow \text{Max. profit} \\
\text{s.t. :} \quad & \forall t \in \mathcal{T}: \\
& y(t) \leq u(t) + b(t-1), && \rightarrow \text{Limited supply} \\
& b(t) = b(t-1) - y(t) + u(t), && \rightarrow \text{Inventory Management} \\
\text{vars. :} \quad & y(t) \geq 0, t \in \mathcal{T}.
\end{aligned}
$$

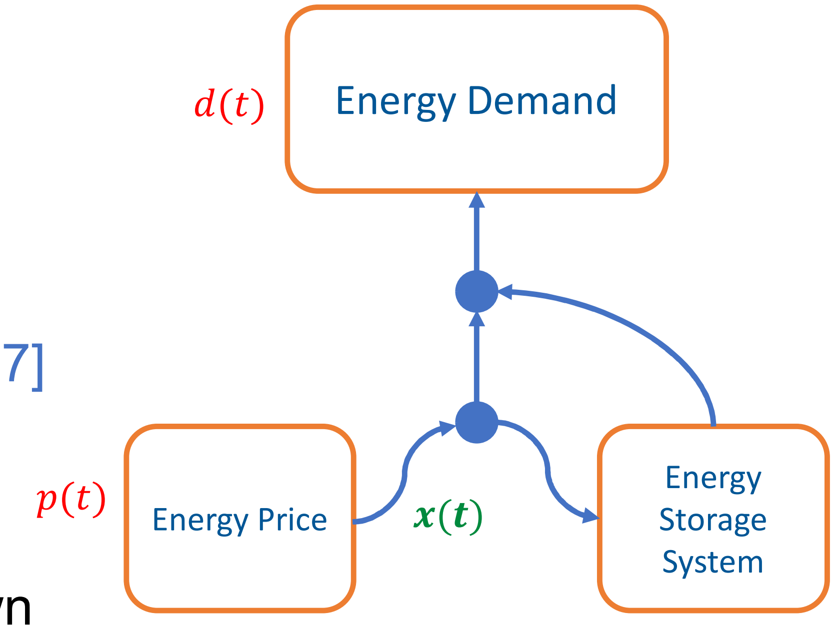
Existing Results

- Extended versions of the k-min and k-max search problems

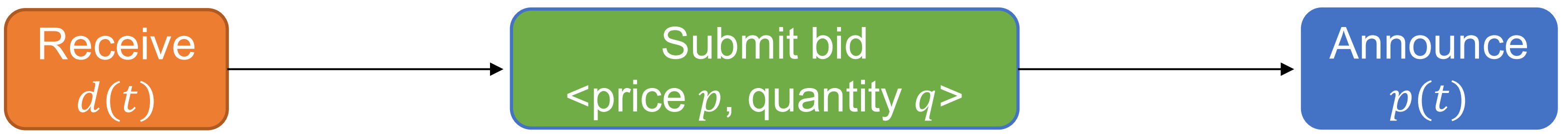
[J. Lorenz, K. Panagiotou, A. Steger, 2008]

- Optimal competitive ratios for cost minimization [L. Yang, et al., Sigmetrics 2020] and profit maximization [L. Yang, M. Hajiesmaili, H. Yi, M. Chen, Sigmetrics 2017]

- **Application:** energy storage and the electricity market.
- In practice, current prices may not be known – **previous algorithms won't work!**
  - Submit bids on energy



Single Bid Scenario



If bidding price $p \geq p(t)$, buy q quantity
at **clearing price** $\boldsymbol{p(t)}$

- Current price $p(t)$ is not immediately known
- Submit an offering bid for price p, quantity q
  - Accept if $p \geq p(t)$, reject otherwise

Multiple Bid Scenario

For all bidding prices $p_i \geq p(t)$, buy q_i quantity at clearing price $p(t)$

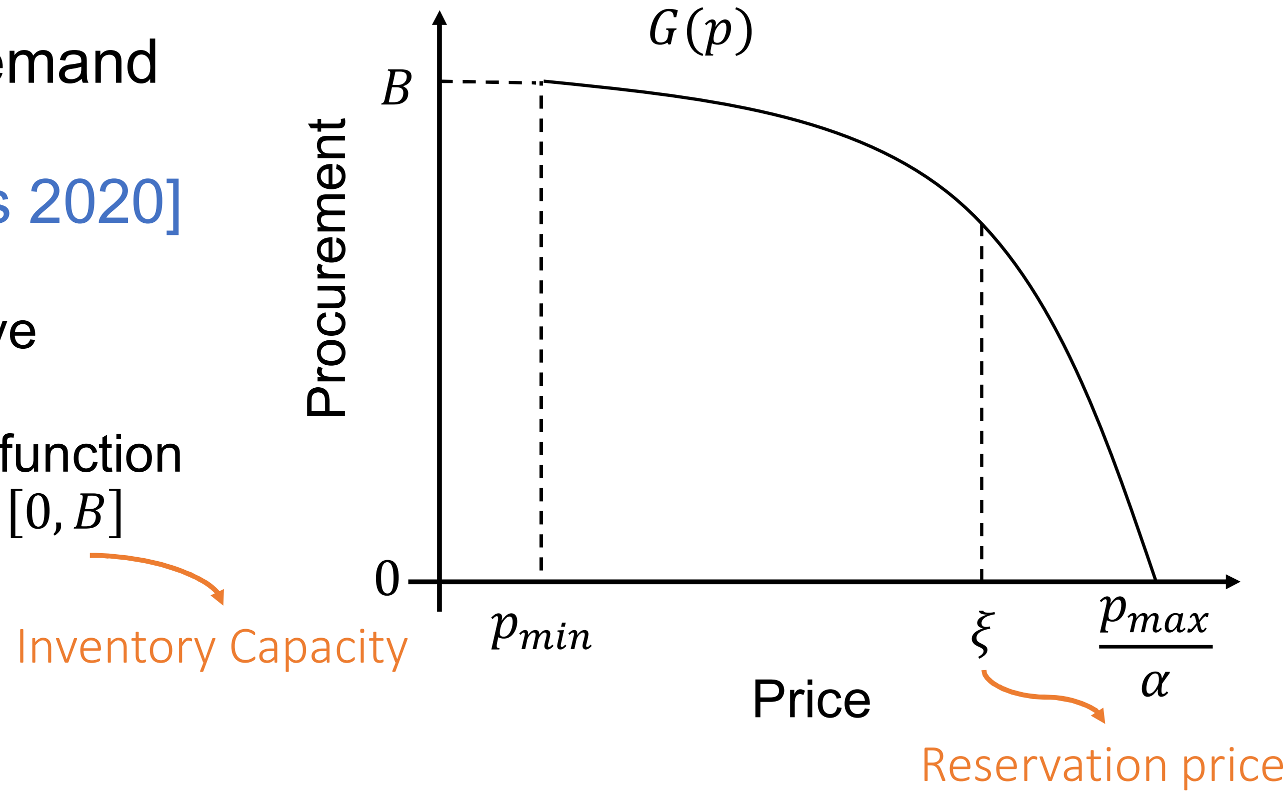
- Current price $p(t)$ is not immediately known
- May submit up to m bids on the asset
  - The i-th bid includes $<$ price p_i, quantity q_i $>$
  - Set of bids $\{< p_1, q_1 >, \dots, < p_m, q_m >\}$
- Bidding strategy design: **be close to online optimal** knowing $p(t)$

An Existing Threshold Function

- The bid-free online demand algorithm (DEM-ON)

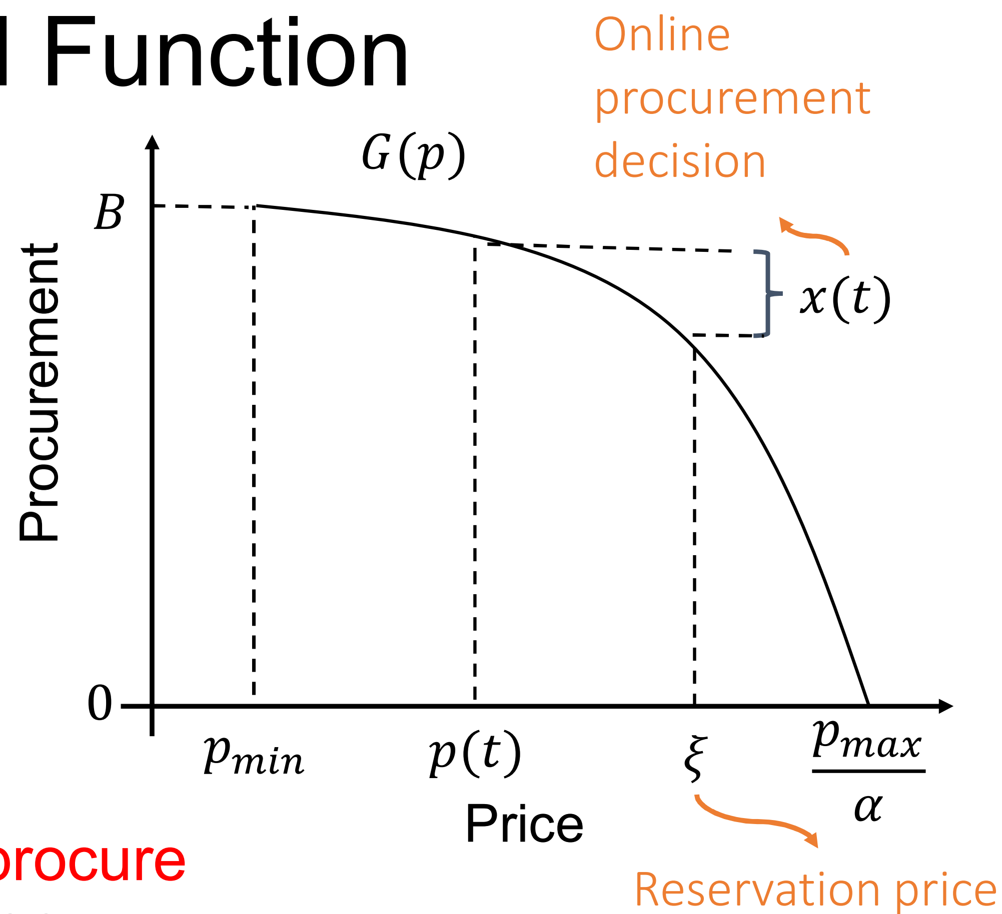
[Yang, et al., Sigmetrics 2020]

  - Optimally α-competitive
  - Decreasing threshold function $G(p): [p_{min}, p_{max}/\alpha\ ] \rightarrow [0, B]$
  - Reservation price ξ



An Existing Threshold Function

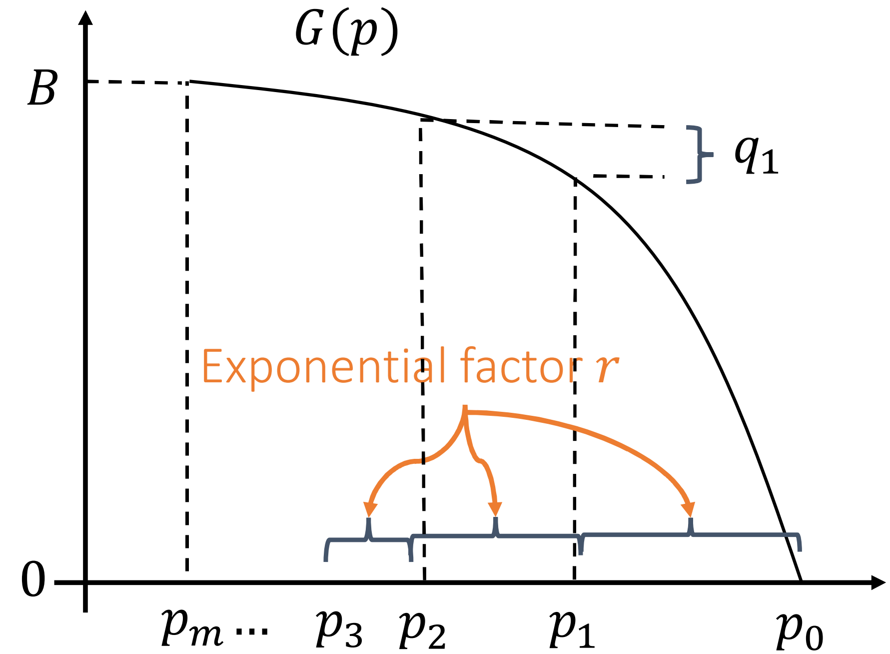
- At each time t, if $p(t) < \xi$:
  - Determine $x(t) = G\big(p(t)\big) - \xi$
    - Cover demand as necessary
  - Update $\xi = p(t)$
    - Reset $\xi = \frac{p_{max}}{\alpha}$ when $b(t) = 0$



- For the bidding setting, want to procure as closely as possible to $x(t)$ even when $p(t)$ not known

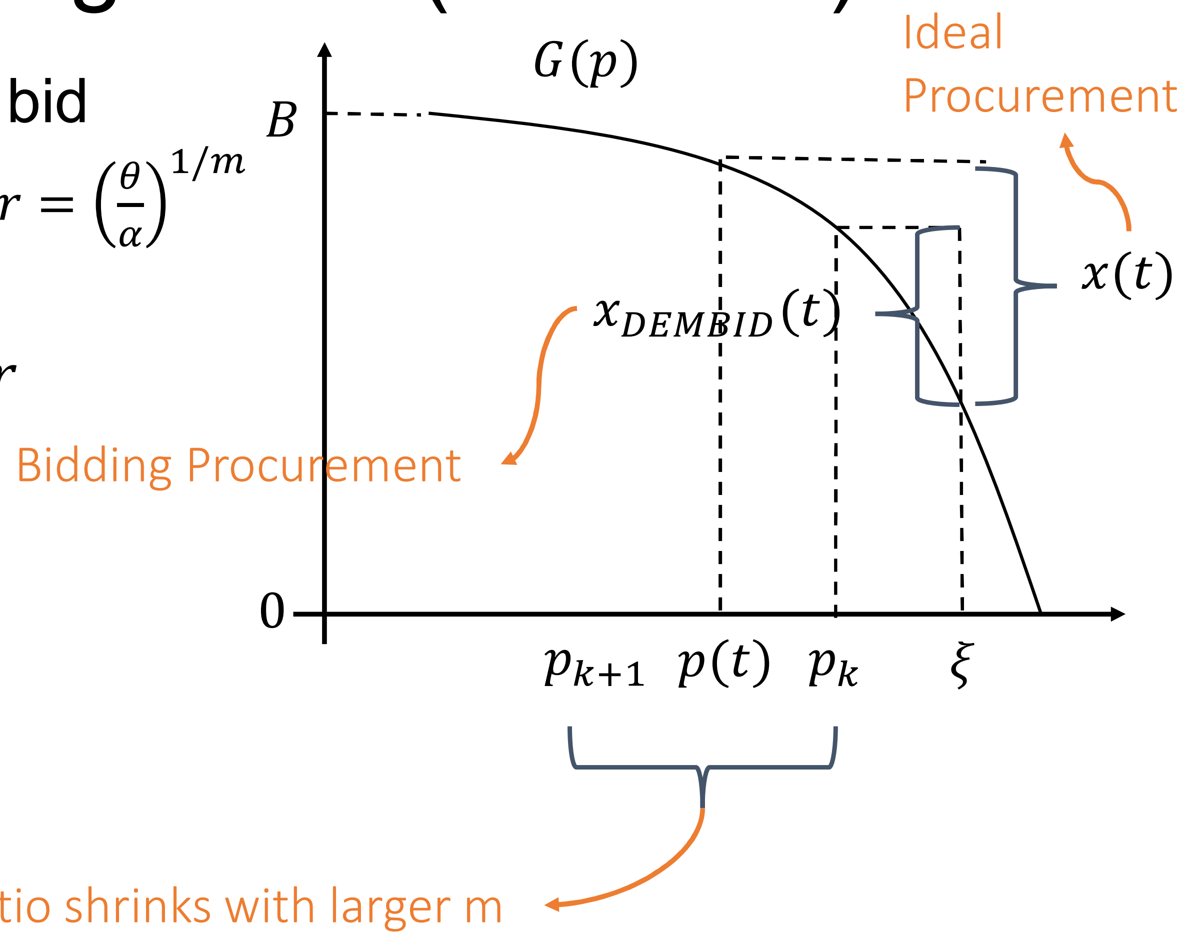
Bid Partitioning

- Set a partition ratio $r = \left(\frac{\theta}{\alpha}\right)^{1/m} > 1$
  - θ is a problem-specific parameter
- Set $p_0 = \frac{p_{max}}{\alpha}$
- At each time t, submit m bids for $i \in [1, m]$:
  - $p_i = \frac{p_0}{r^i}$
  - $q_i = [G(p_i) - G(\min(p_{i-1}, \xi))]^+$



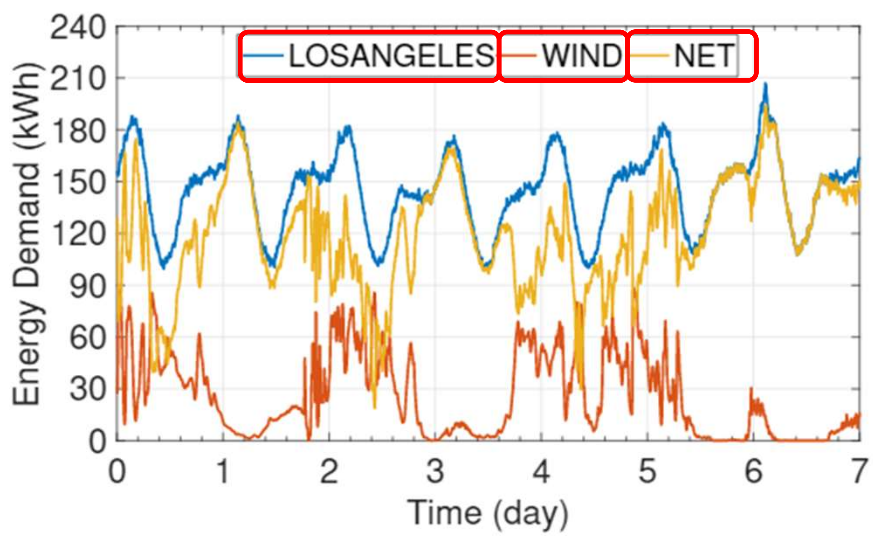
Demand Bidding Algorithm (DEMBID)

- p_k is the largest accepted bid
  - Partitioning ensures $\frac{p_k}{p(t)} \leq r = \left(\frac{\theta}{\alpha}\right)^{1/m}$
- **Theorem**: DEMBID is $\alpha \cdot r$ competitive
  - $r \to 1$ as $m \to \infty$
  - Approaches optimal online competitive ratio α



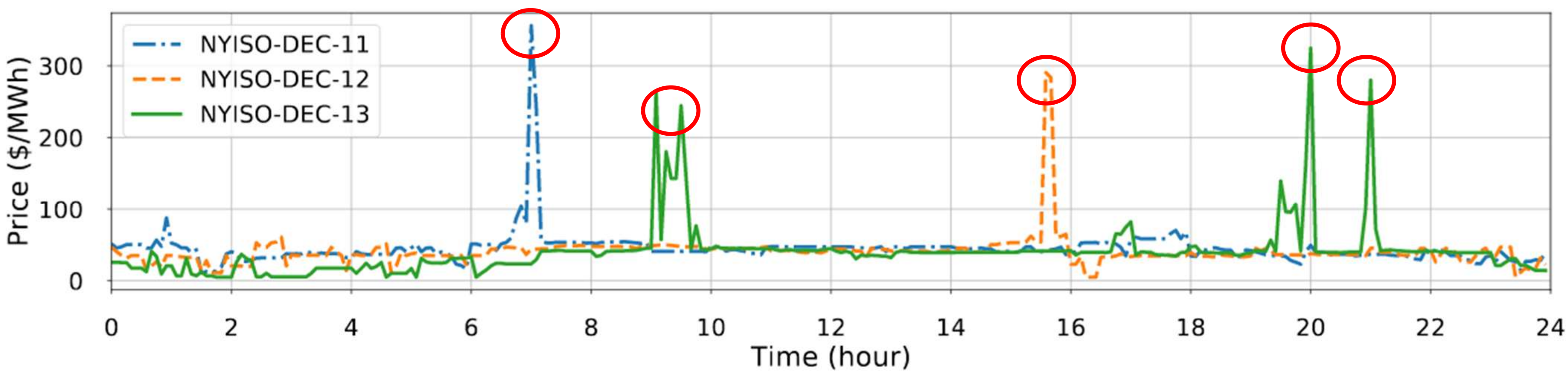
Data Traces and Experimental Setup

- Data center energy demand
  - 338 Akamai data centers
  - Renewable generation data from National Renewable Energy Laboratory (NREL)
- Unpredictable renewables leads to unpredictable net demand



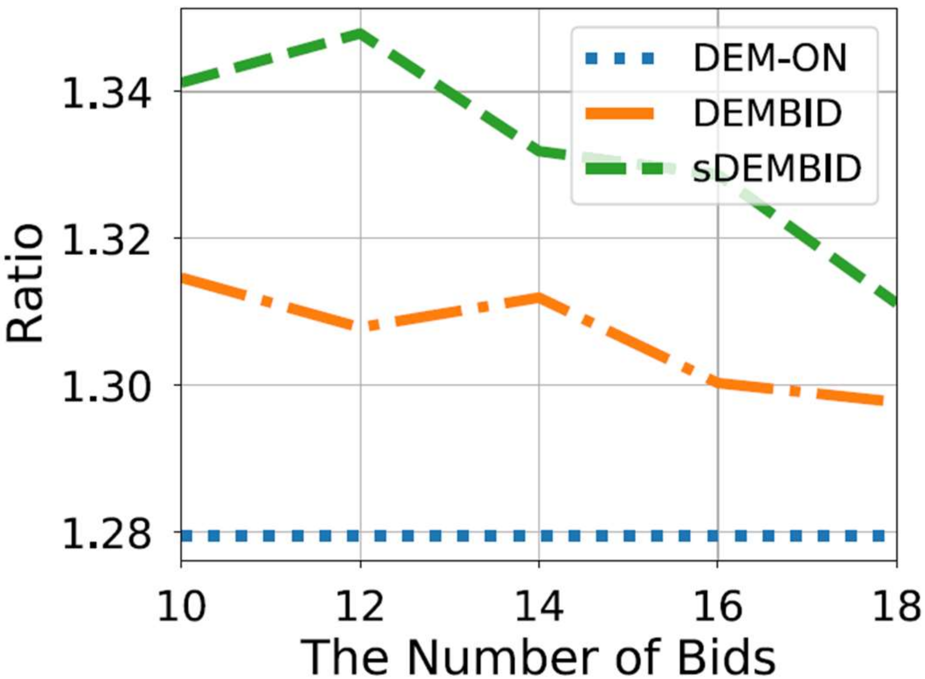
Data Traces and Experimental Setup

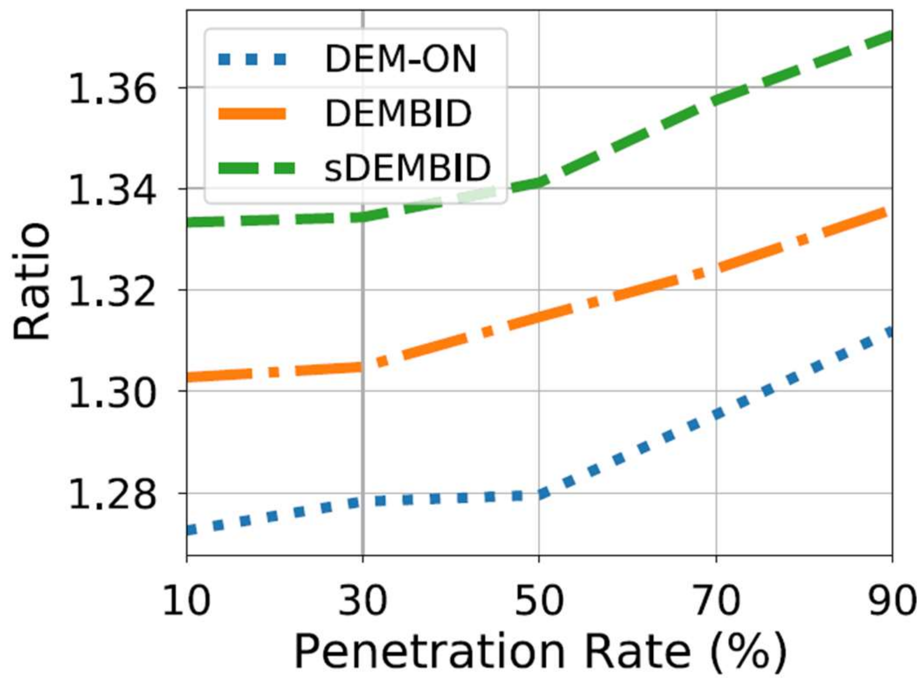
- Energy prices
  - 2018 spot energy prices from NYISO
  - 5 minute time slots
  - Unpredictable energy prices over 3 different days



Experimental Results

- sDEMBID – simplified algorithm





Conclusion

- **Designed competitive bidding strategies** to deal with uncertainty in online inputs.
- The competitive ratio of DEMBID **approaches the optimal online competitive ratio** with increasing bids.
- Future direction
  - Use data from other locations to evaluate performance on different energy markets