# Optimal Speed Profile of a DVFS Processor under Soft Deadlines

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# Presentation of the problem

- Characteristics of the processor:
  - DVFS processor (Dynamic Voltage and Frequency Scaling) working at variable speed $s$.
  - Power dissipation of the processor $w(s)$.
- Characteristics of the jobs:
  - Jobs arrive randomly, with random sizes and deadlines.
  - Jobs that are not completed before the deadline are dropped from the queue, and induce a cost $K_{miss}$.
- Objective: Find the optimal speeds that minimize the long term energy spent by the processor and the cost of missed deadlines.

# System description

*Processor:* A DVFS processor with speed $s \in [0, S_{max}]$, and the power dissipation $w$ is an increasing, strictly convex function (classic model: $w(s) = s^3$).

*Jobs:* Poisson arrival process with rate $\lambda$
Deadlines: exponential law with parameter $\delta$
Sizes: exponential law with parameter $\mu$.

$\Rightarrow$ We can formulate the problem as an MDP.

# Rates of the MDP

We consider the continuous-time MDP: $(\mathbb{N}, [0, S_{max}], Q, c)$.
Let $\sigma = (\sigma_i)_{i \in \mathbb{N}}$ be a stationary speed policy:

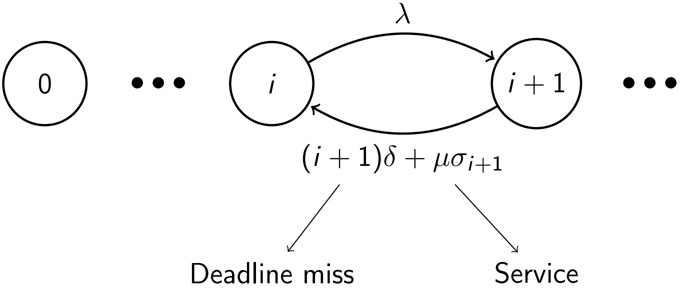



Figure: Markov chain $X^\sigma$ under policy $\sigma$.

# Cost function

Define $c(\cdot,\cdot)$ as the expected cost endured by the system at time $t$:

$$c(i,\sigma) := K_{miss} i\delta + w(\sigma_i).$$

For any policy $\sigma$, $X^\sigma$ is ergodic.
$\Rightarrow$ The long term cost $J(\sigma)$ exists and the optimal cost satisfies the Bellman optimality equation.

# Main result

## Theorem

*There exists a deterministic optimal policy $\sigma^* = (\sigma_i^*)_{i\in\mathbb{N}}$ that is increasing in $i$ and upper bounded by $B$, where:*

$$B := \arg\min_{s\in\mathbb{R}^+} (w(s) - K_{miss}\mu s).$$

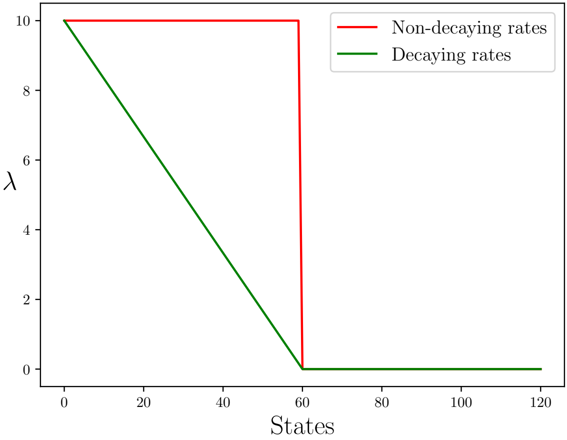
For example: with $w(s) = s^3$ (i.e., the classic model), $B = \sqrt{\frac{\mu K_{miss}}{3}}$.

Three key steps in the proof:

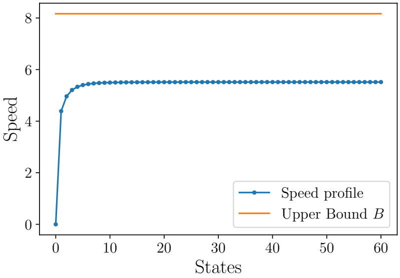
- Truncate the MDP
- Prove the result in the truncated case
- Take the limit in the optimality equation

# First step: Truncation of the MDP

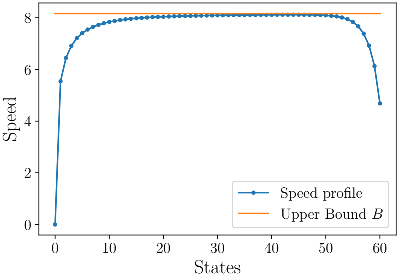
The former MDP cannot be uniformized. A solution is to truncate it.



# Decaying vs no decay: figures




(a) $\mathcal{M}_N$ with decaying arrival rates




(b) $\mathcal{M}'_N$ without decay of the arrival rates

Figure: Optimal policies $\sigma^*$ and $\sigma^{*'}$ for $\mathcal{M}_N$ and $\mathcal{M}'_N$.

## Second step: Lemma for the truncated case

Let

$$B^N := \arg\min_{s\in\mathbb{R}^+}\left(w(s) - \frac{K_{miss}\mu s}{1+\frac{\lambda}{\delta N}}\right).$$

Then,

### Lemma

*The optimal speed policy $\sigma^*$ is:*

(i) ***unique***.

(ii) ***increasing** in $i$: $\forall i \leq N, \sigma_i^* < \sigma_{i+1}^*$.*

(iii) ***upper-bounded**: $\forall i \leq N, \sigma_i^* \leq B^N$.*

# Third step: Taking the limit

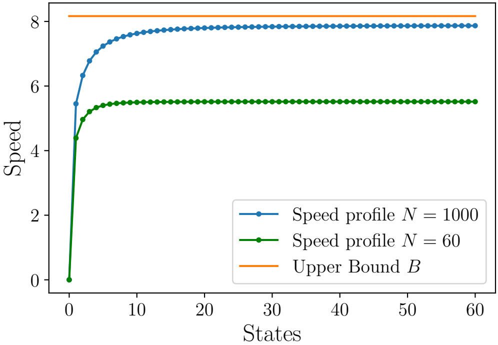



Figure: Two optimal speed policies in $\mathcal{M}_N$, for $N = 60$ and $N = 1000$ and the bound $B = \sqrt{\frac{\mu K_{miss}}{3}}$.

# Approximating the average cost

Let $\sigma^B$ be the policy such that: $\sigma_i^B = B\,\mathbb{1}_{\{i>0\}}$, and $\mu = 1$.

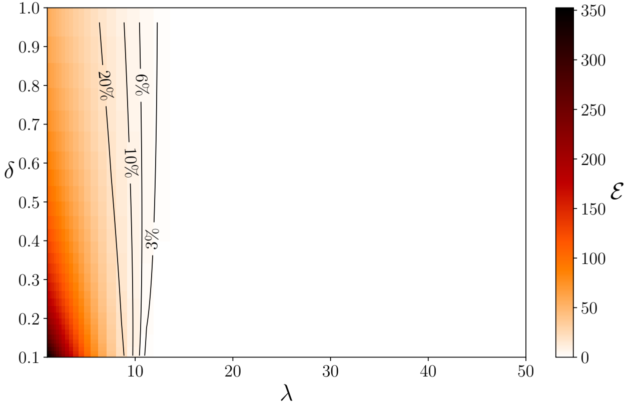



Figure: Levels sets of the percentage relative error $\mathcal{E}$ with a fixed cost per deadline, $K_{miss} = 300$.

# Deadline miss probability

$K_{miss}$ is hard to estimate. We instead look at the probability for jobs to miss their deadline in the stationary regime.

**Definition**

The probability that a job misses its deadline under the stationary regime of $\sigma^B$ is:

$$p_{miss} := \sum_{i \geq 1} \frac{\pi_i^B}{1 - \pi_0^B} \frac{\delta i}{\delta i + B}.$$

# Approximation when $p_{miss}$ is fixed

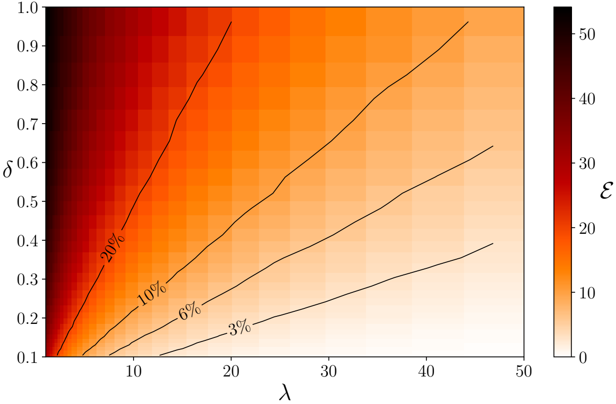



Figure: Levels sets of the percentage relative error $\mathcal{E}$ with a fixed miss probability $p_{miss} = 0.1$.

# Conclusion

- The result holds even if the power dissipation function $w$ is non strictly convex (example: piecewise linear, when the processor works with a finite set of speeds).
- The smooth scaling method is powerful for the analysis of structural properties.
- Future research: reinforcement learning with unknown rates of the MDP.