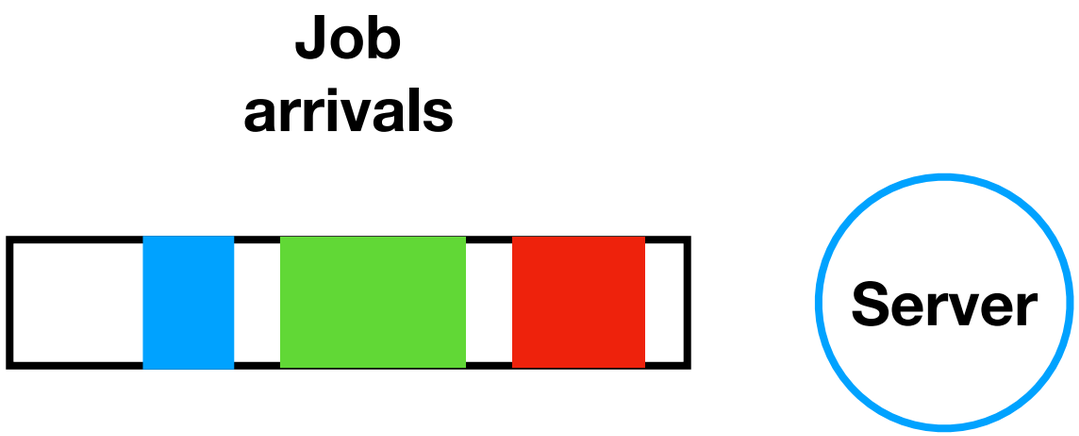
Speed Scaling with Sum-Power Constraint

Rahul Vaze

Tata Institute of Fundamental Research, Mumbai

Joint work Jayakrishnan Nair- IIT-Bombay

Scheduling 101



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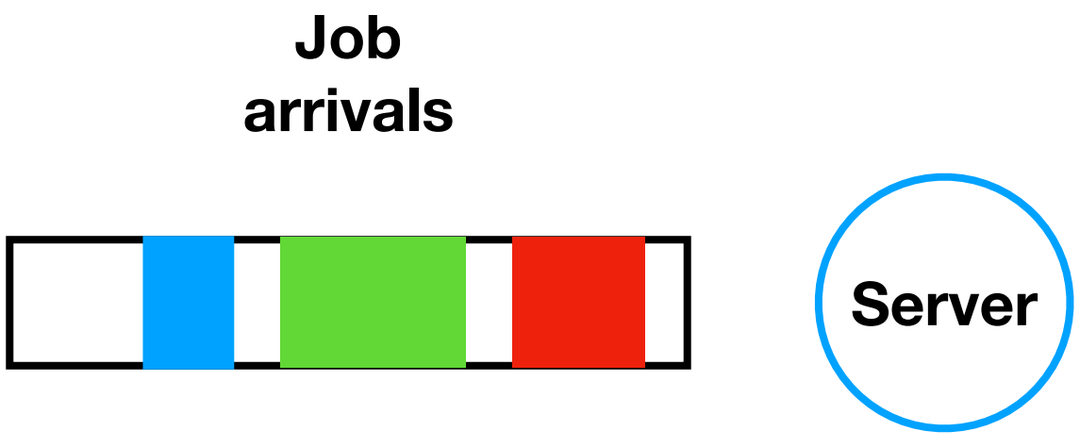

Single server

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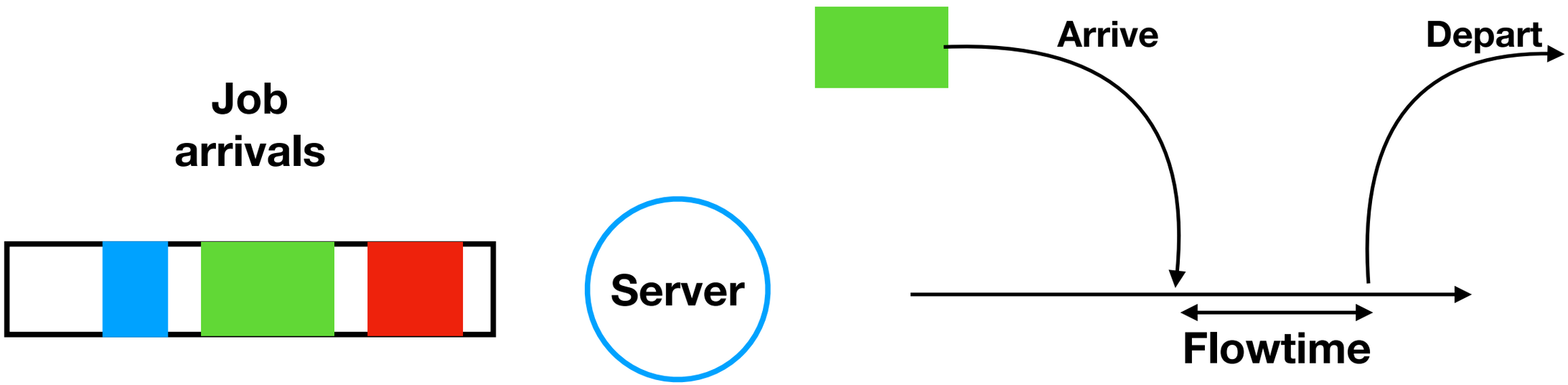

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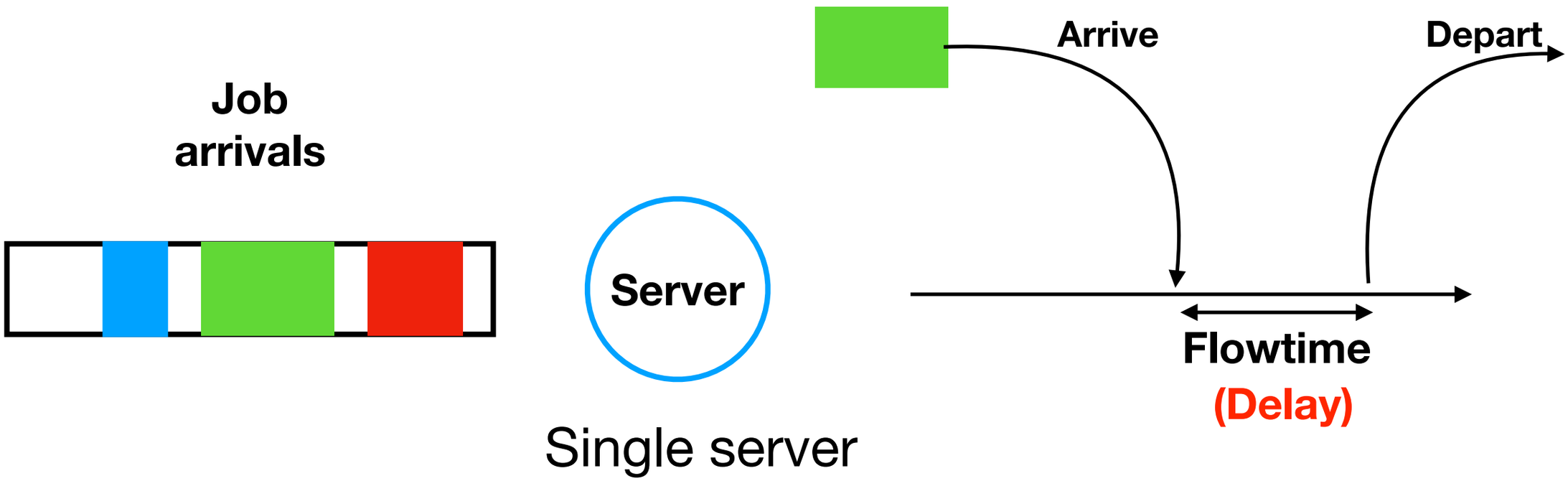

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Obj: *min* **total flow time**

$$\sum_{j \in \text{Jobs}} \text{flow time}_j = \int n(t)dt$$

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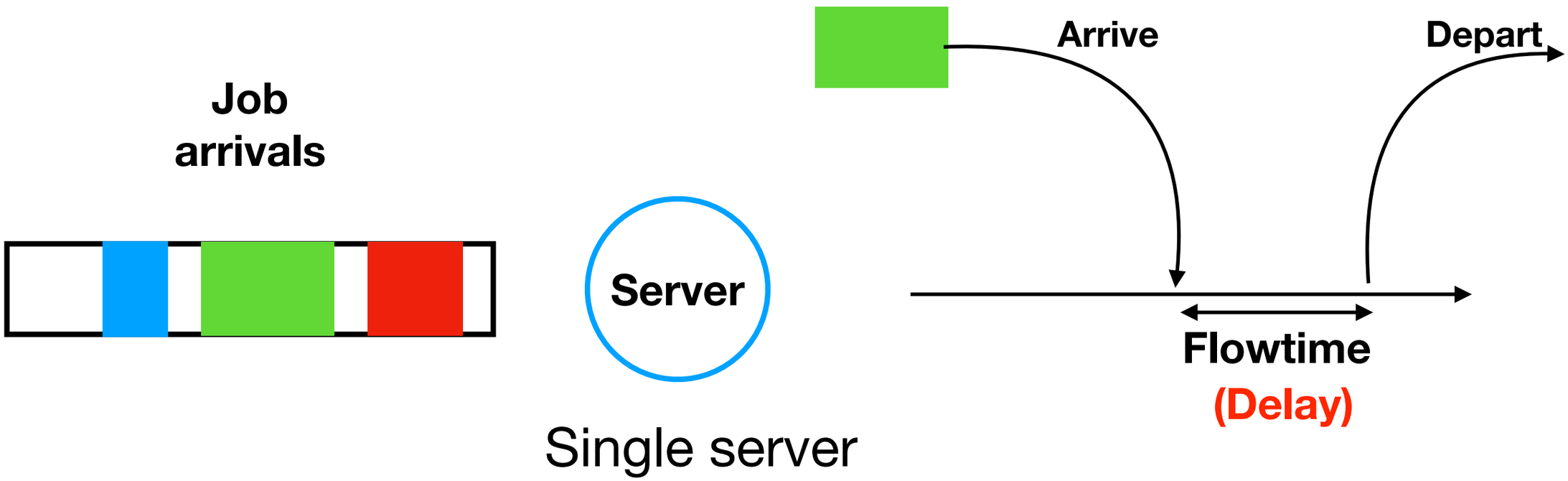

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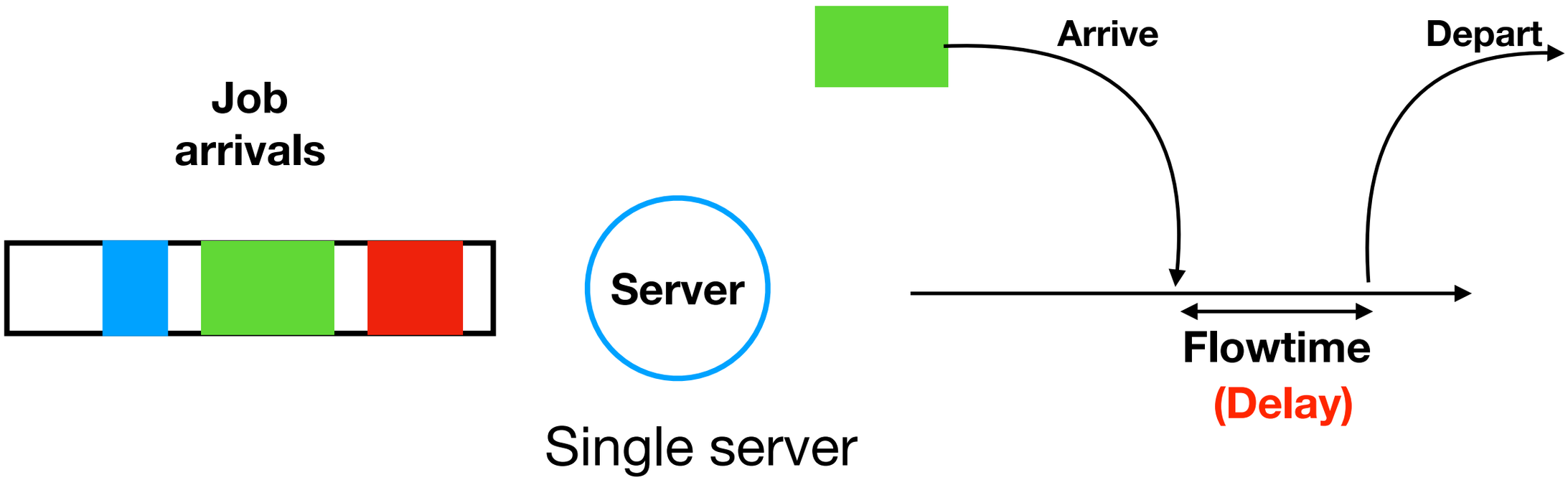

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SRPT is optimal

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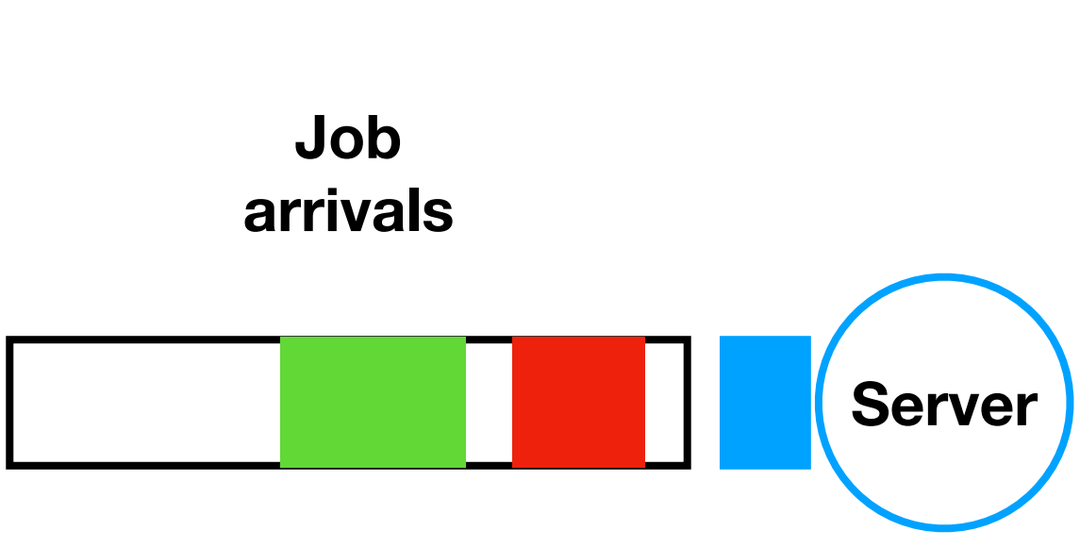

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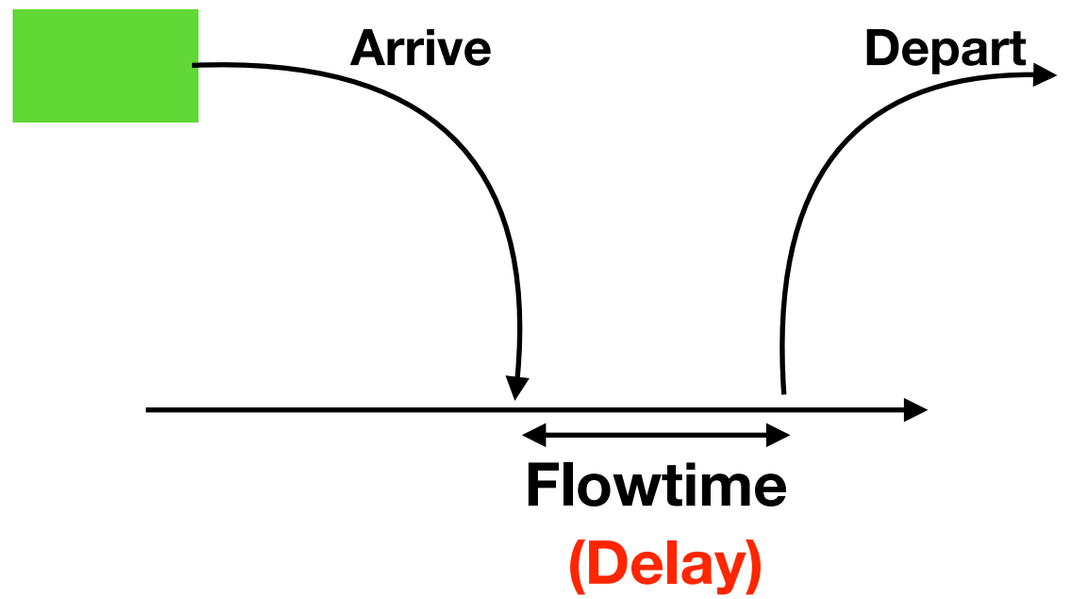

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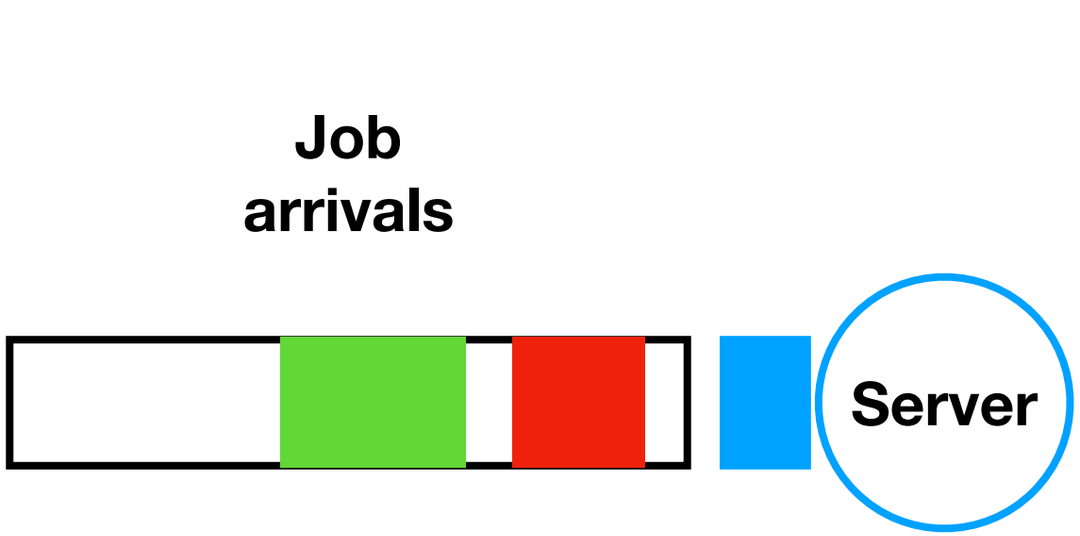

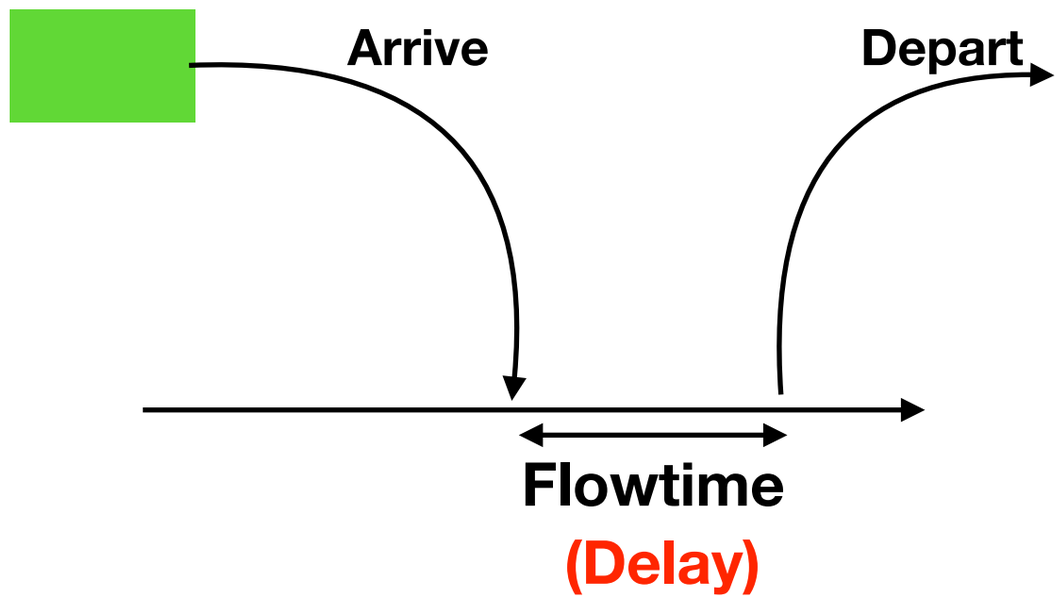

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Parallel Scheduling

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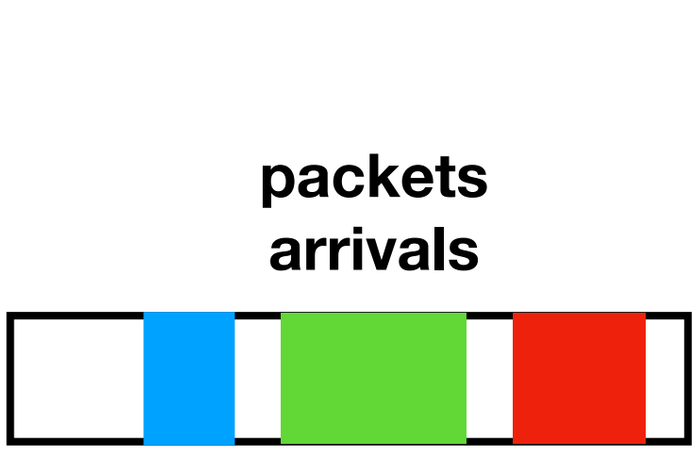

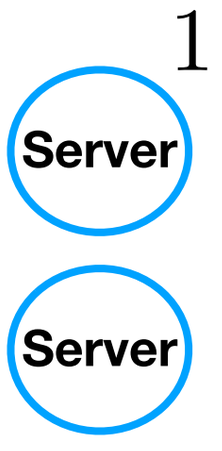

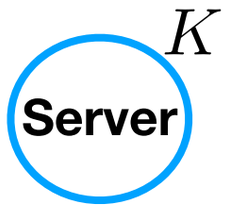

Multiple Servers

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$$\int n(t)dt$$

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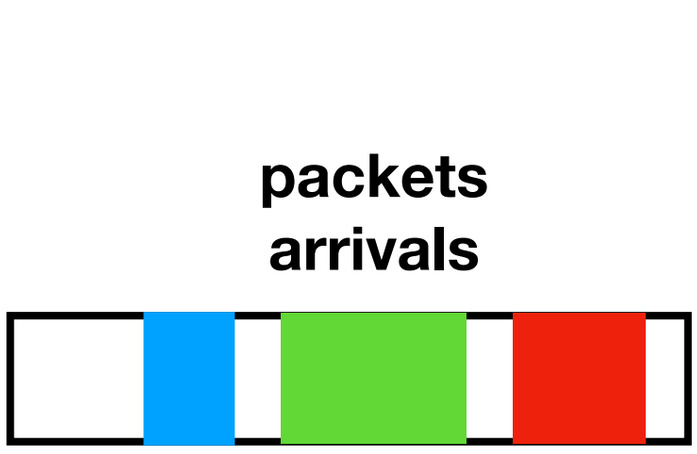
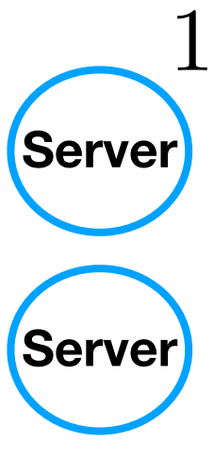
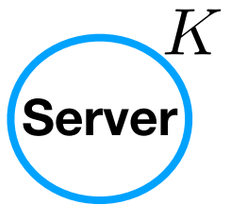
Parallel Scheduling







Multiple Servers

Fully Parallelizable Jobs

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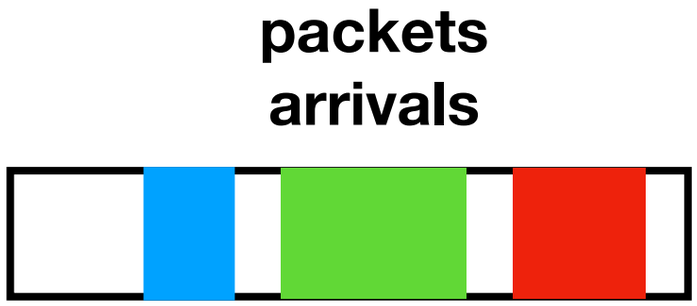
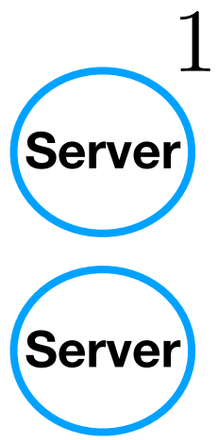
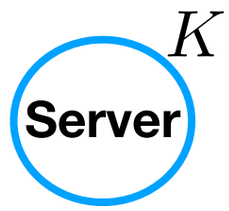
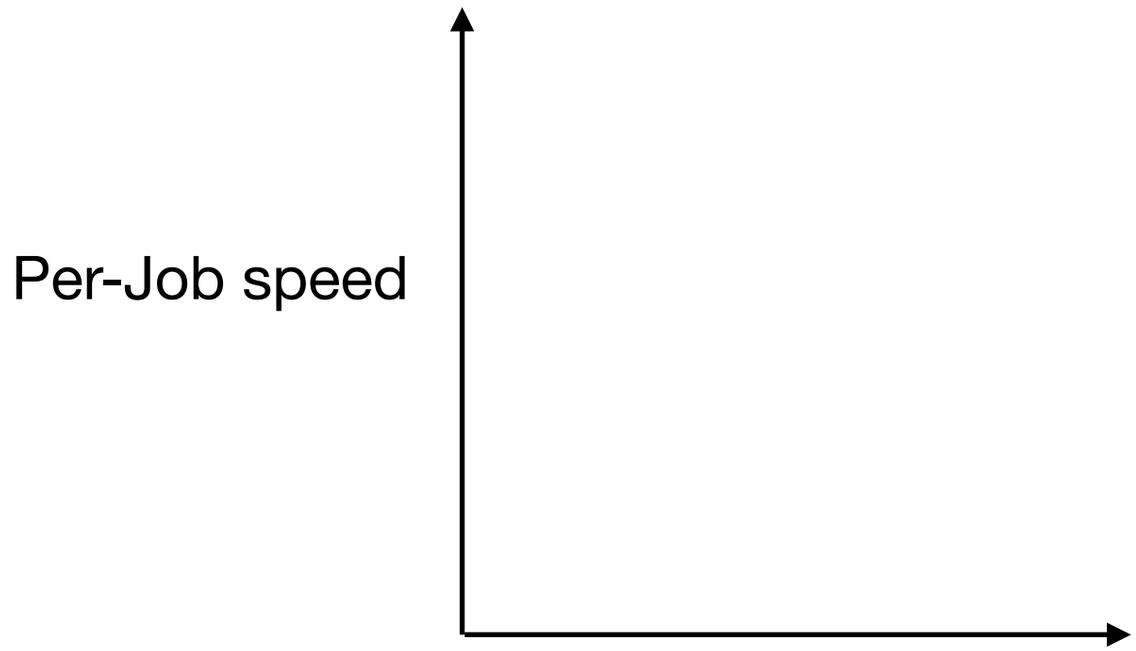






Multiple Servers



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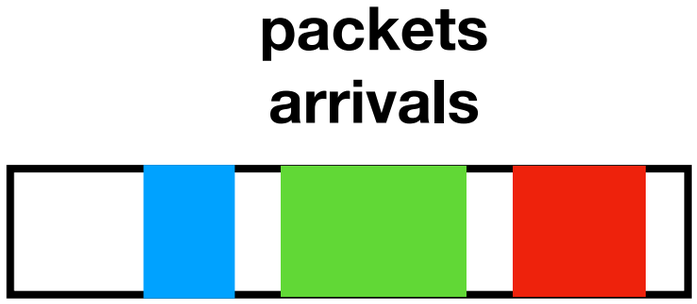

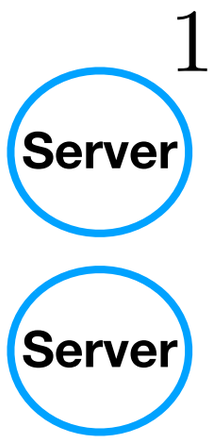

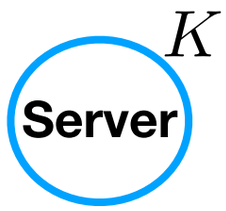

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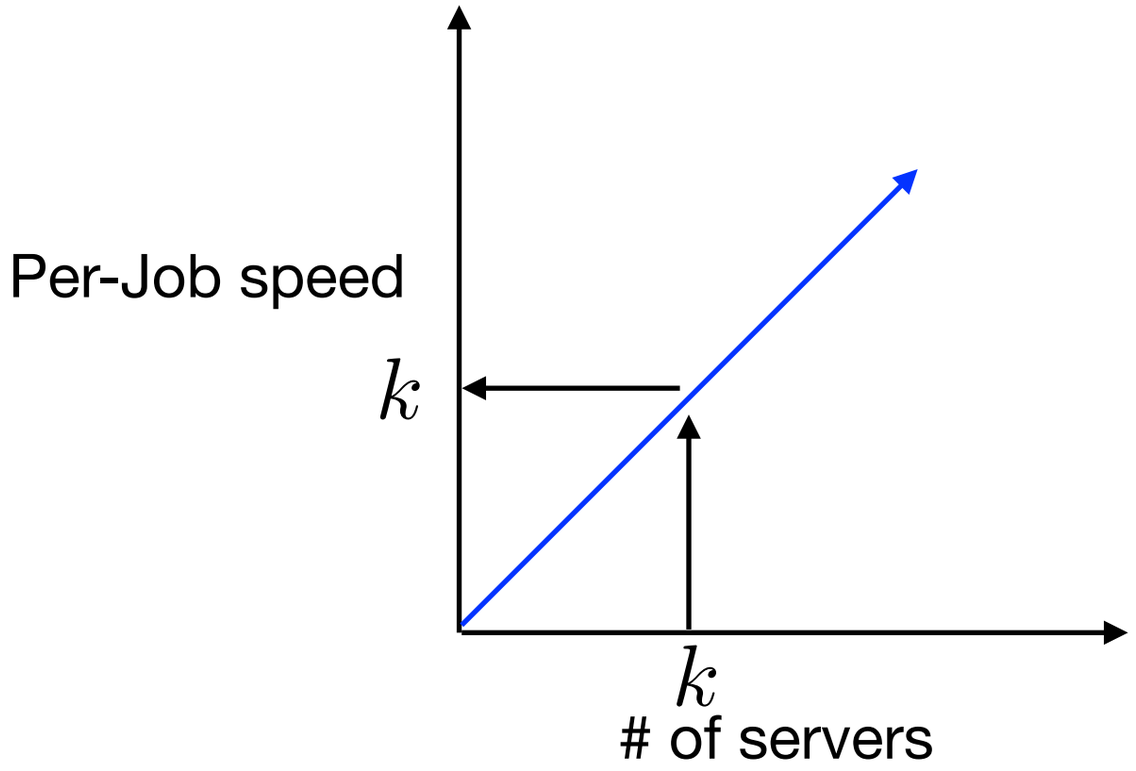

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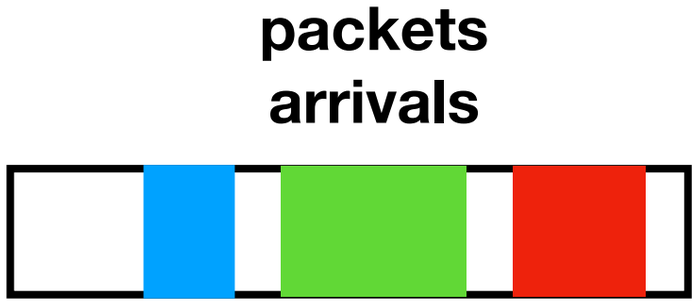

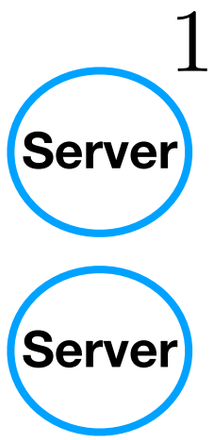

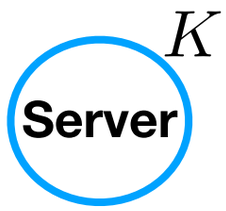

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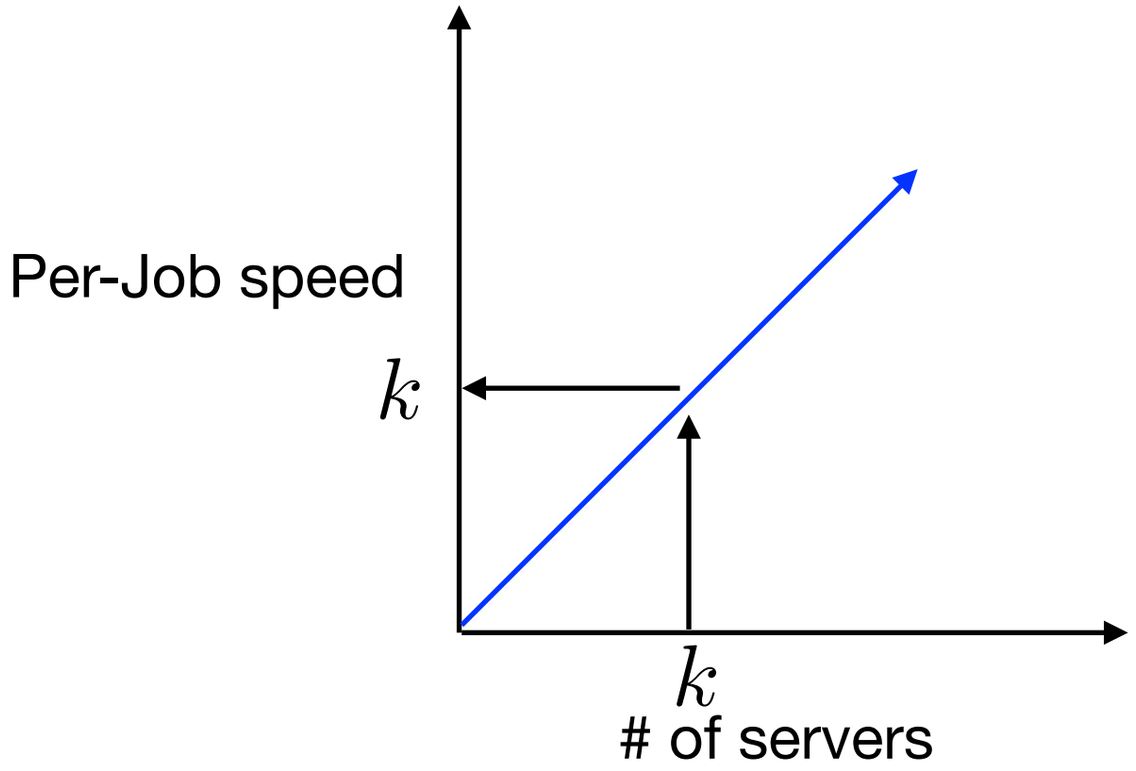

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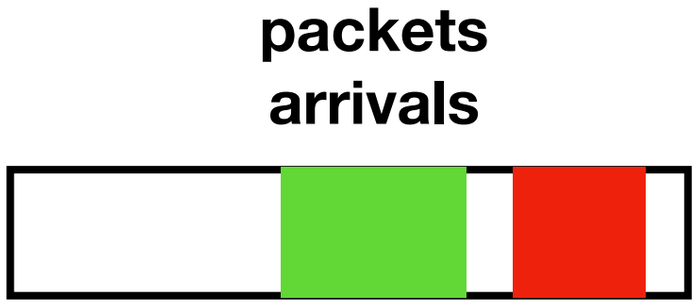

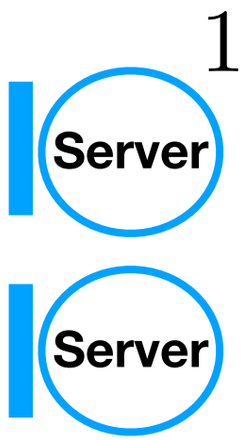

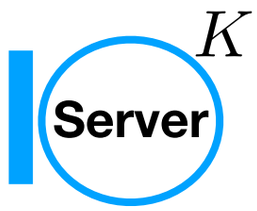

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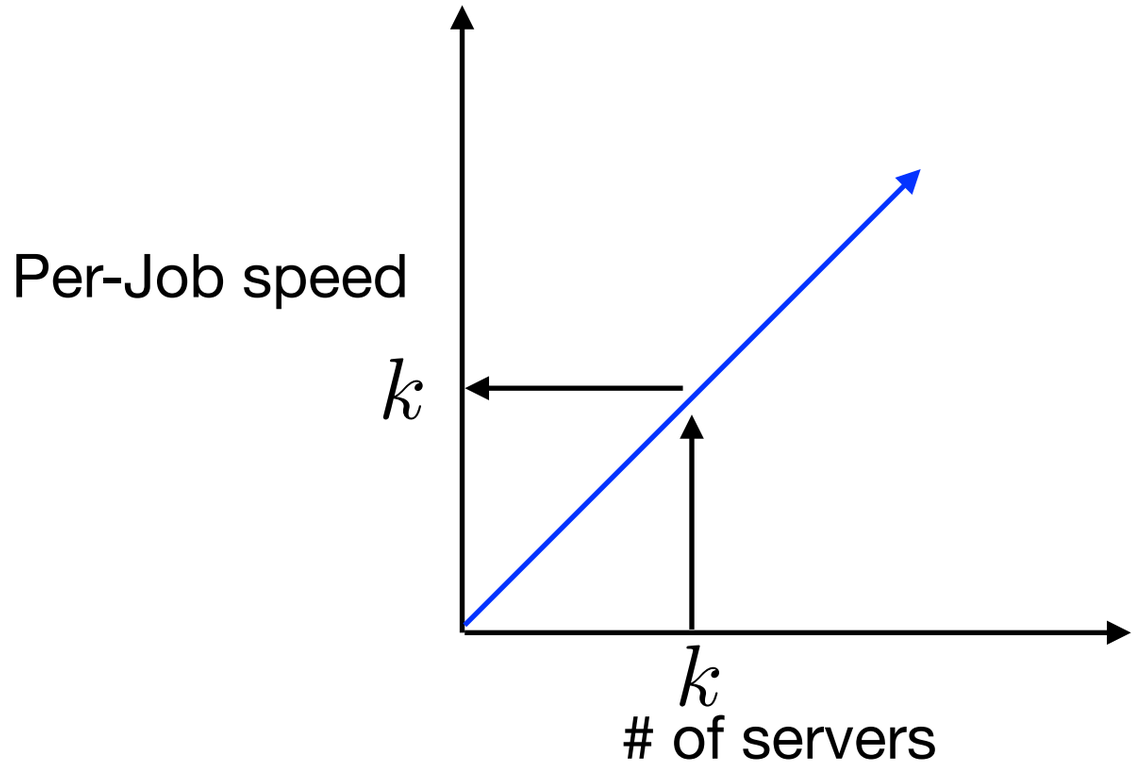

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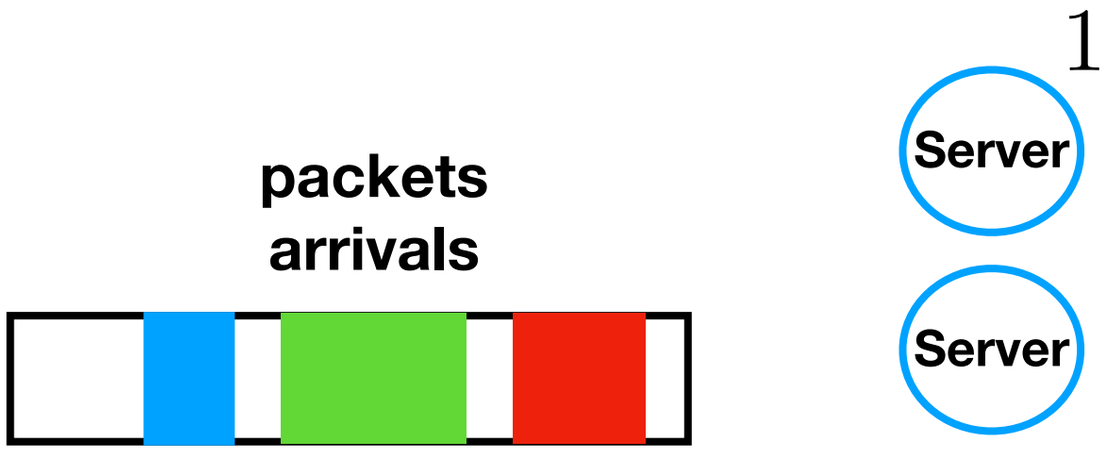
Parallel Scheduling- Reality



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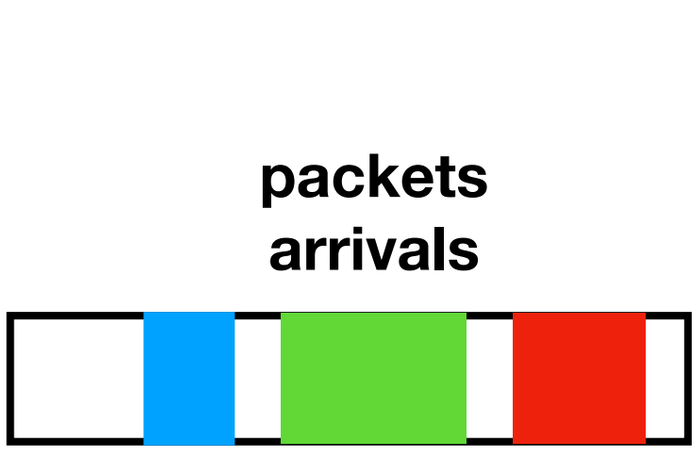

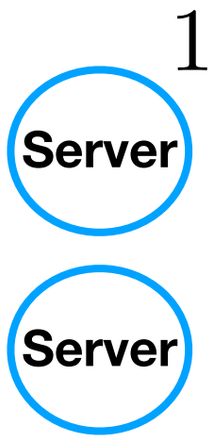

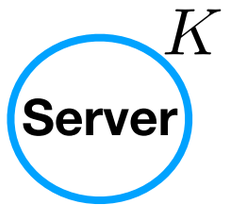

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Limited Parallelizable Jobs

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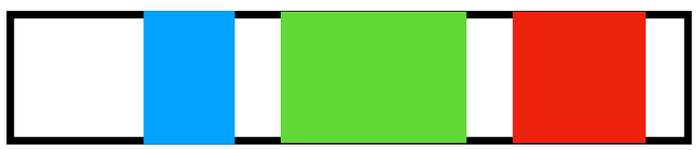

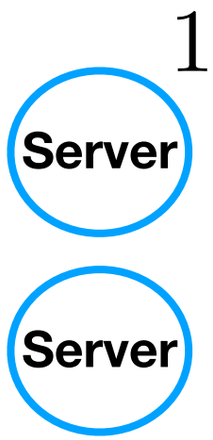

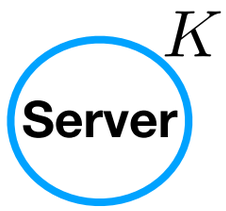

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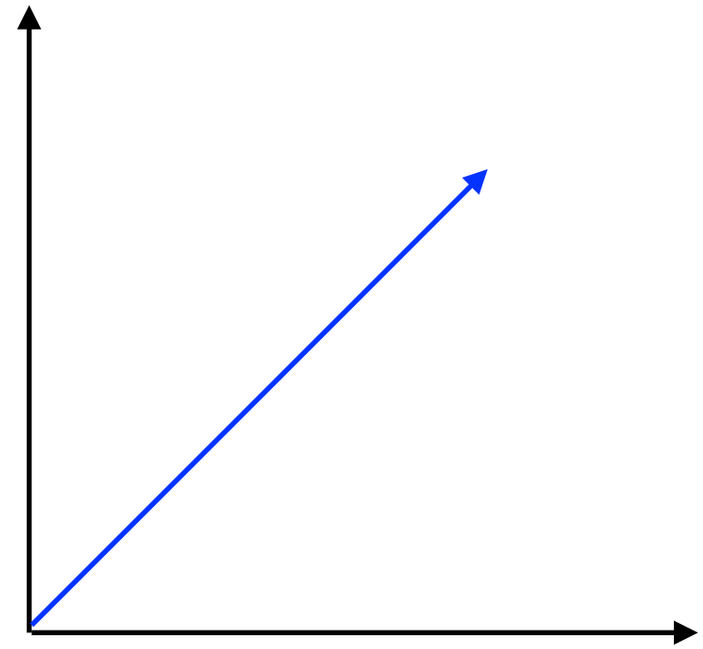

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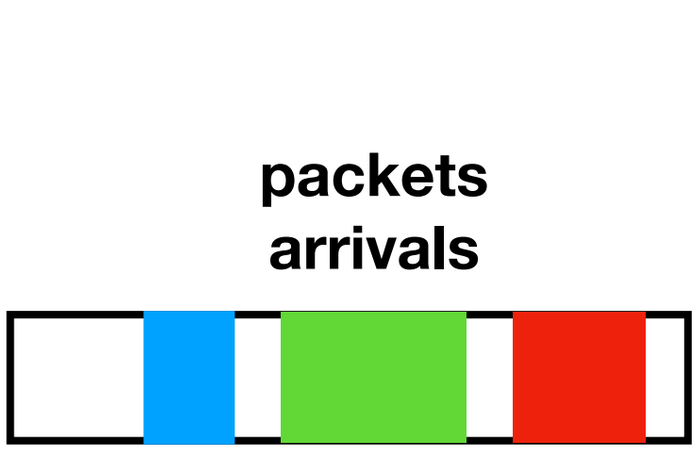

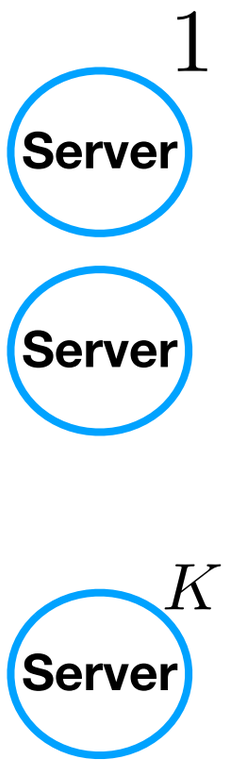

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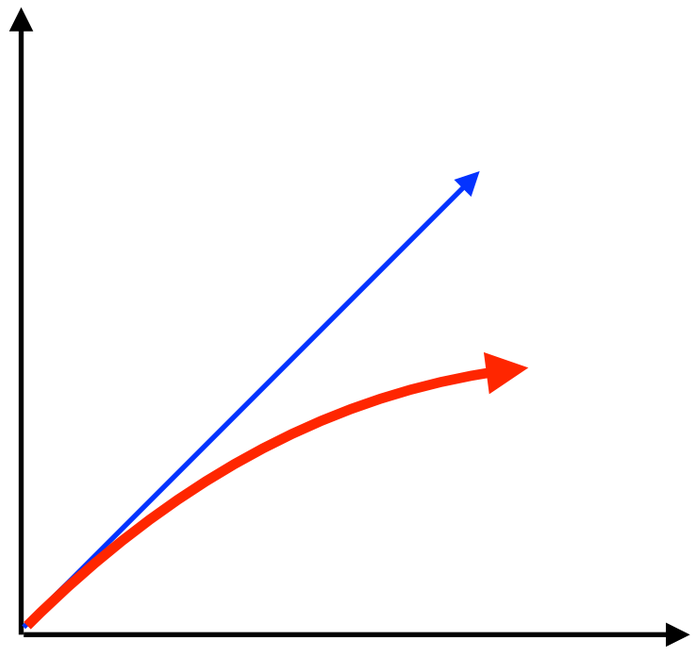

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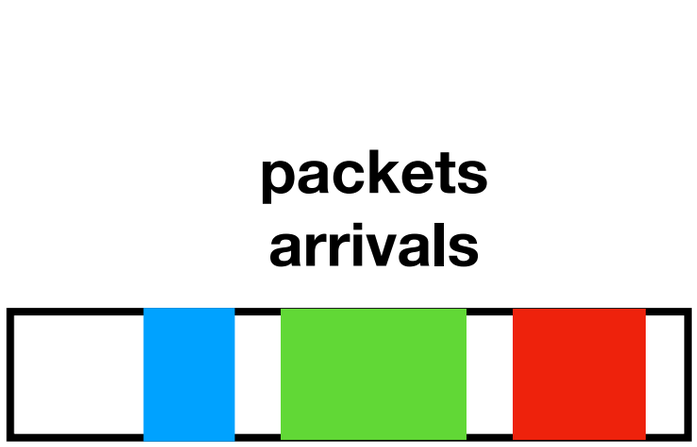

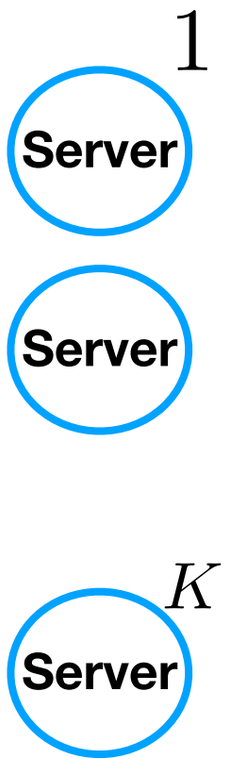

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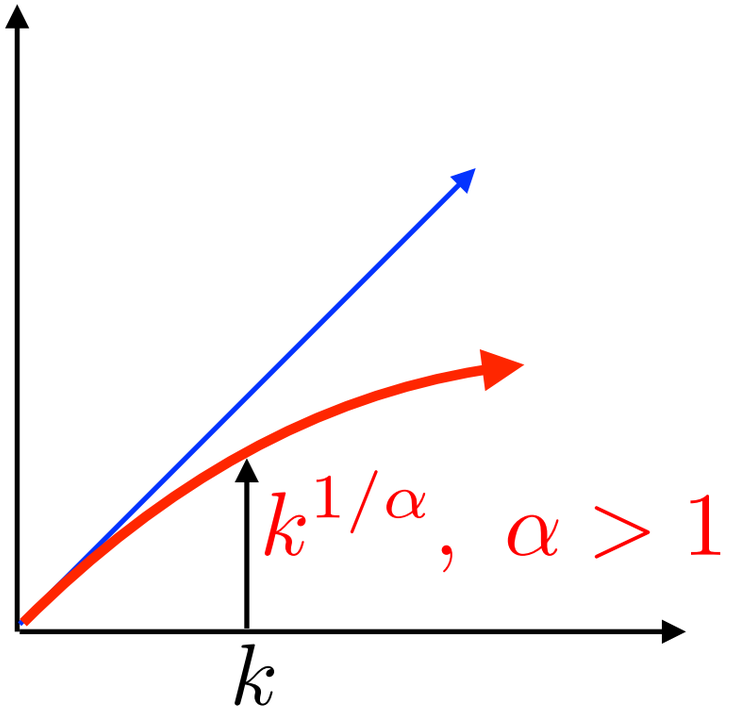

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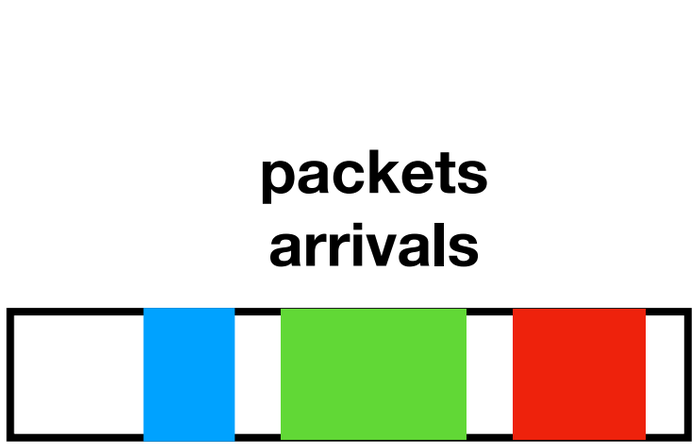

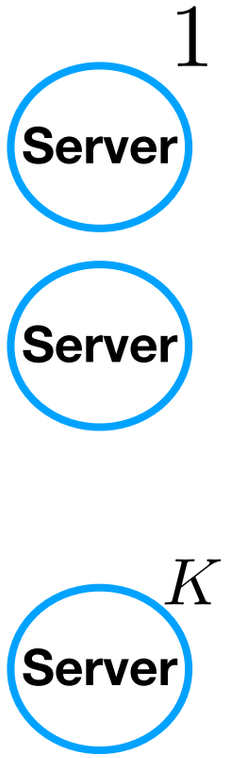

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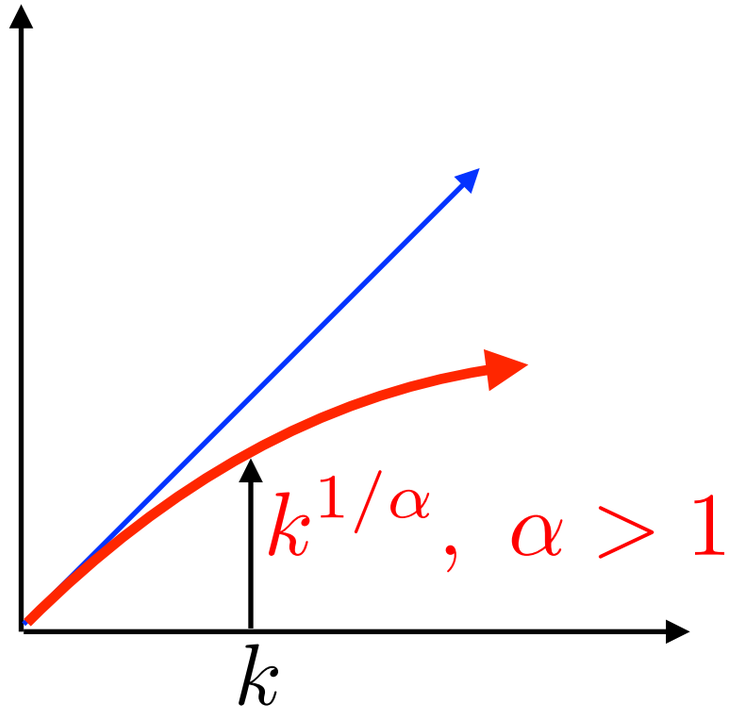

Limited Parallelizable Jobs

Diminishing returns by using more servers for the same job

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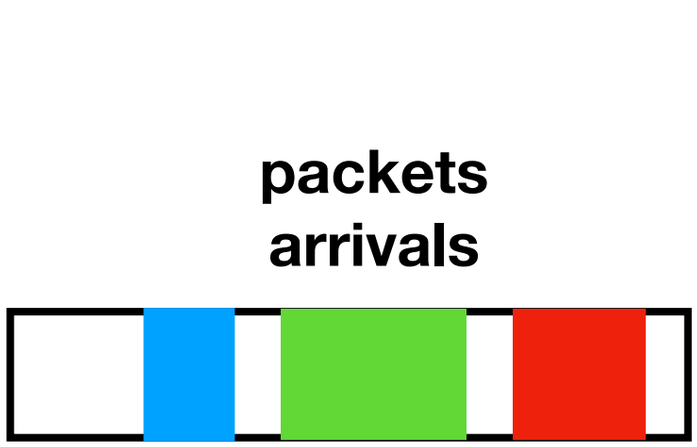
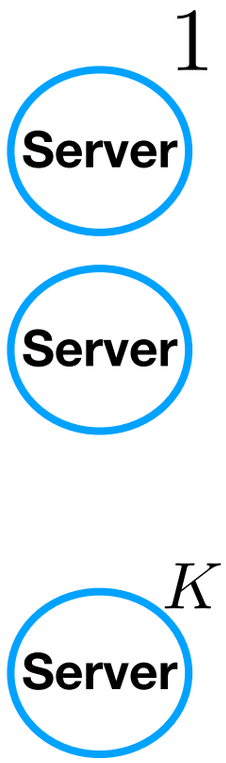
Parallel Scheduling- Reality





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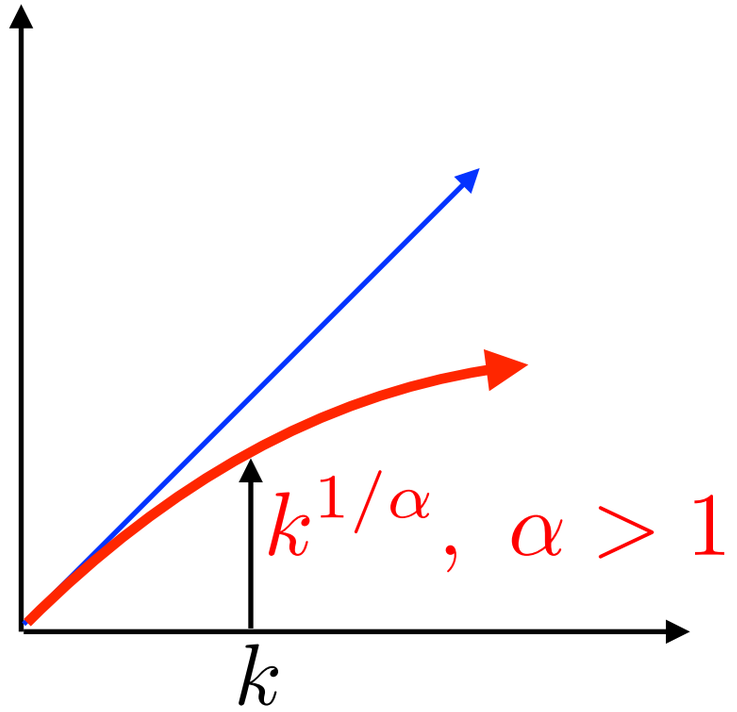

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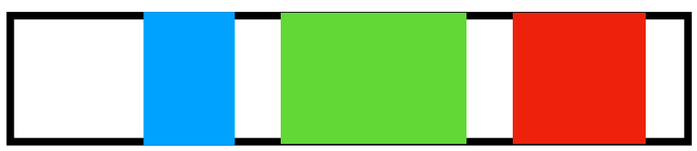
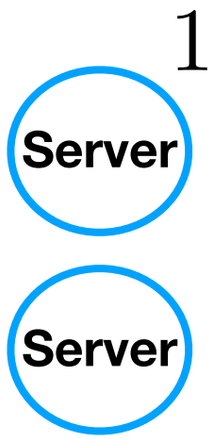
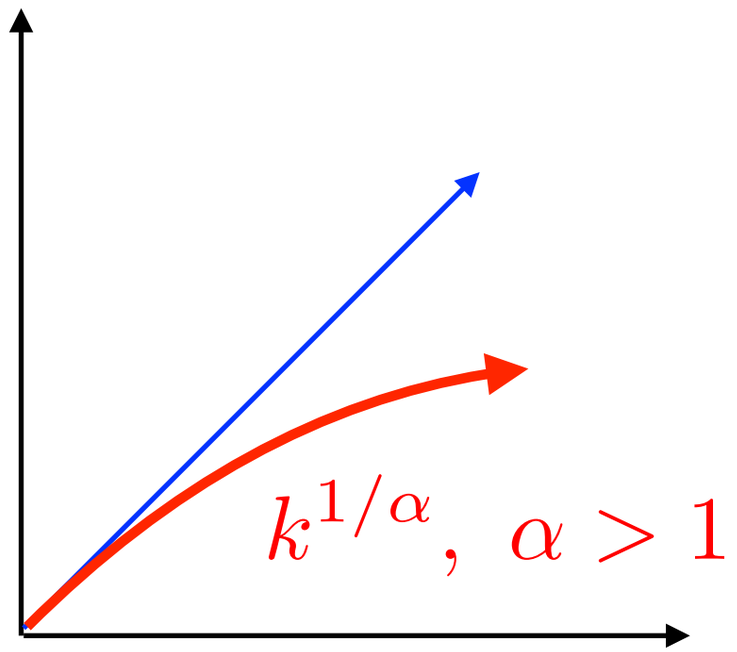
Open Question:
Optimal Scheduling

Prior Work



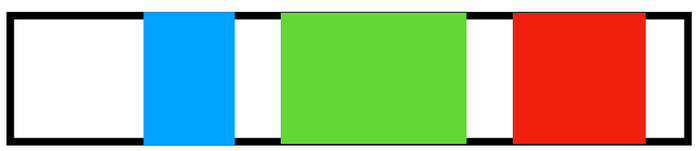


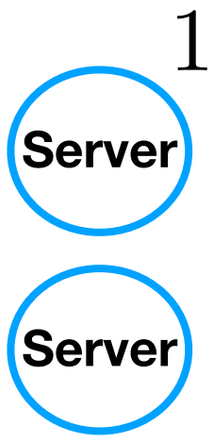
Multiple Servers



**Limited
Parallelizable Jobs**

Prior Work





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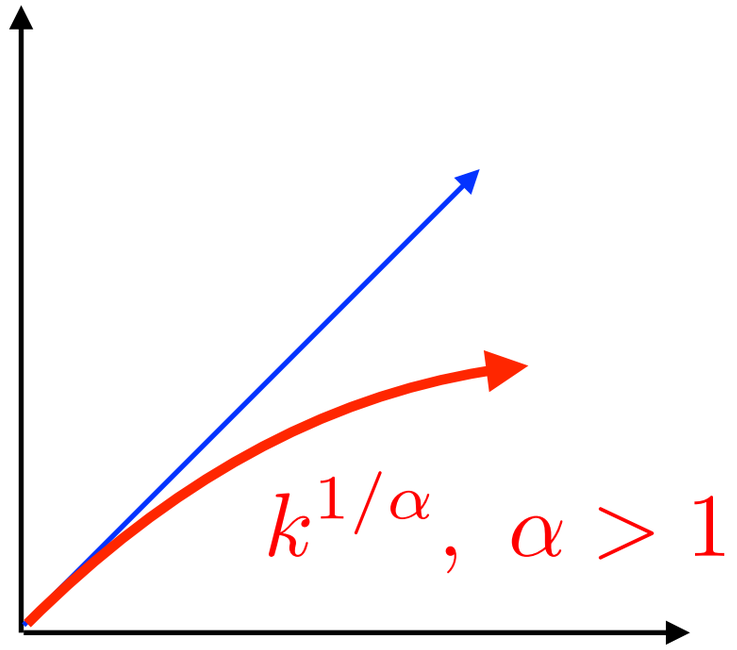
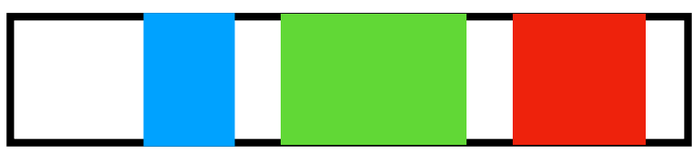
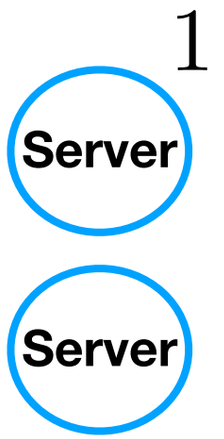


Limited
Parallelizable Jobs

All jobs available at time 0,

Prior Work





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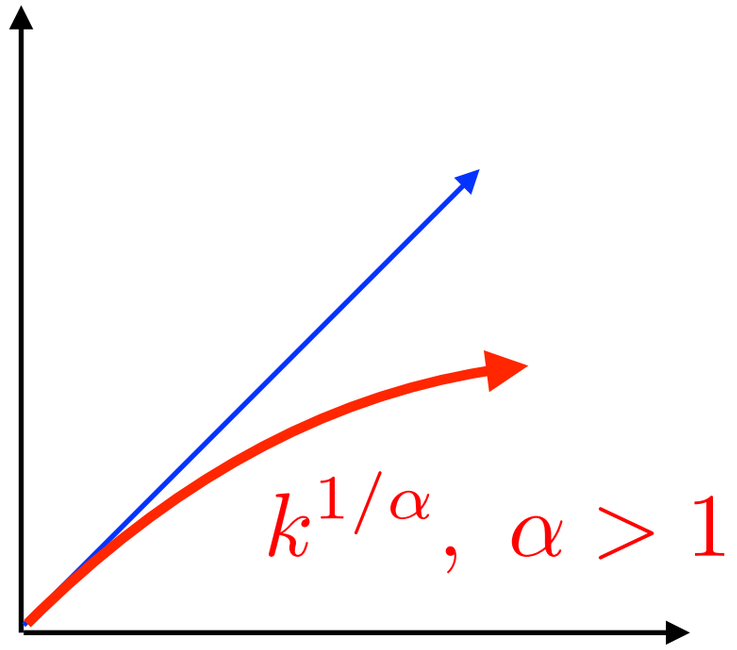
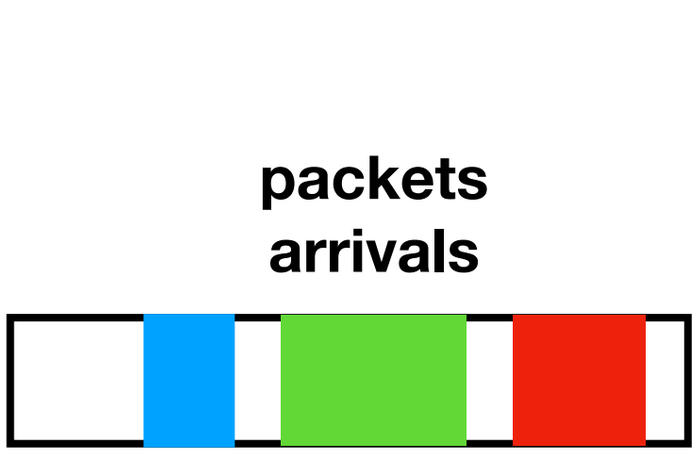
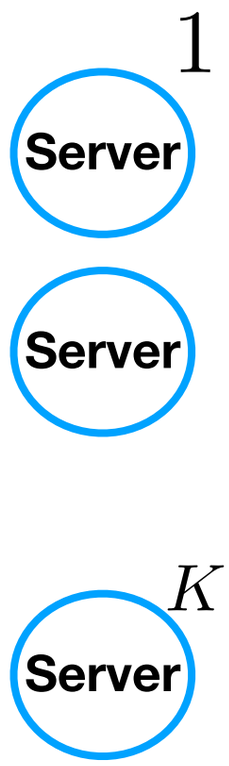


Limited Parallelizable Jobs

All jobs available at time 0, Optimal Scheduling : heSRPT **[Berg et al' 20]**

Prior Work

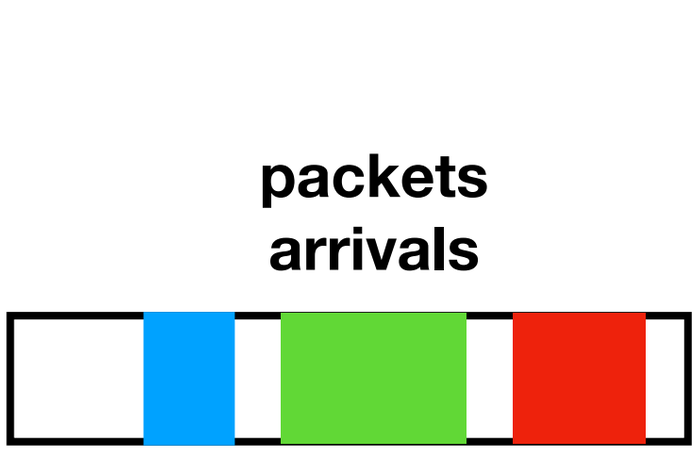


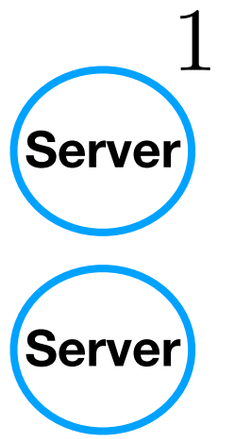


Multiple Servers

All jobs available at time 0, Optimal Scheduling : heSRPT **[Berg et al' 20]**

Prior Work



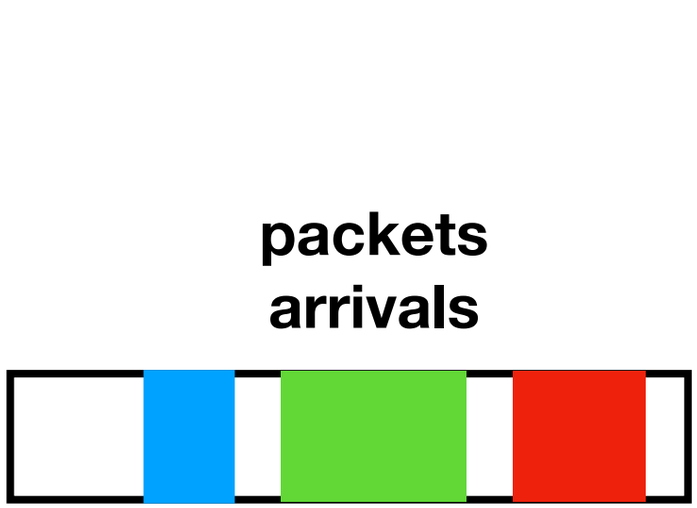


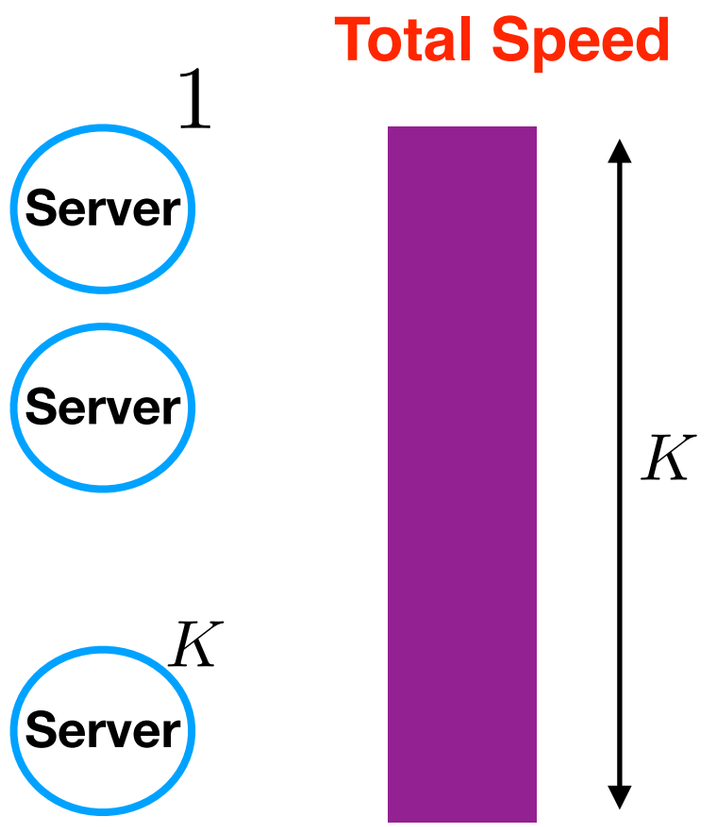
Multiple Servers

All jobs available at time 0, Optimal Scheduling : heSRPT **[Berg et al' 20]**

All jobs get non-zero speed, while shorter jobs get more speed

Prior Work





Multiple Servers

All jobs available at time 0, Optimal Scheduling : `heSRPT` **[Berg et al' 20]**

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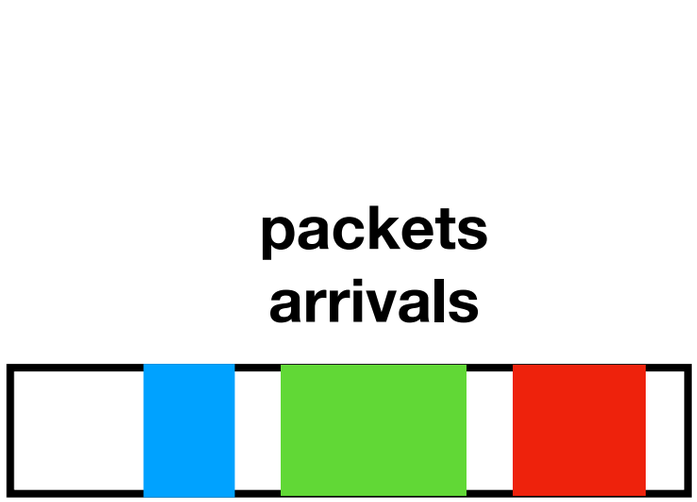
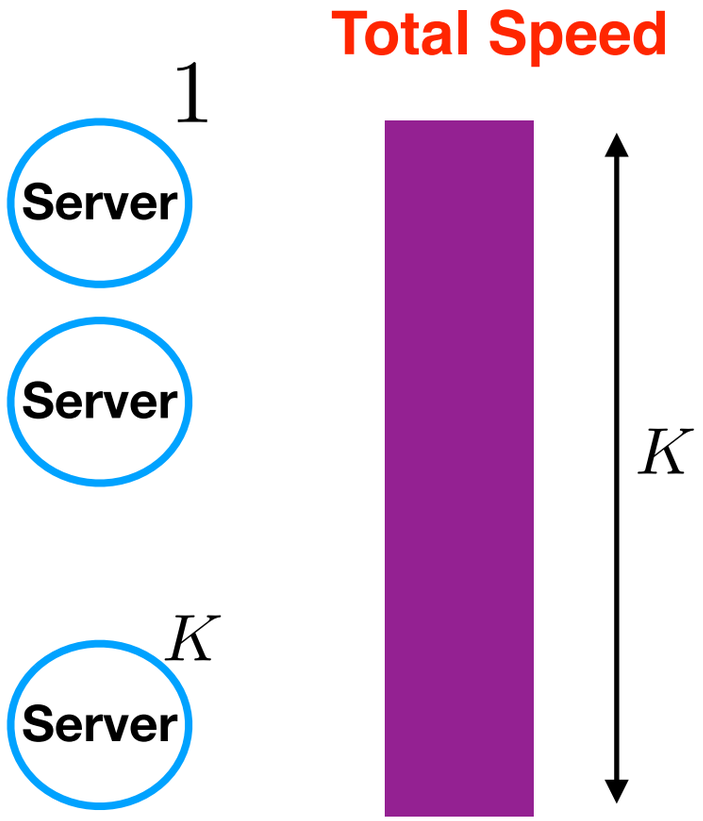
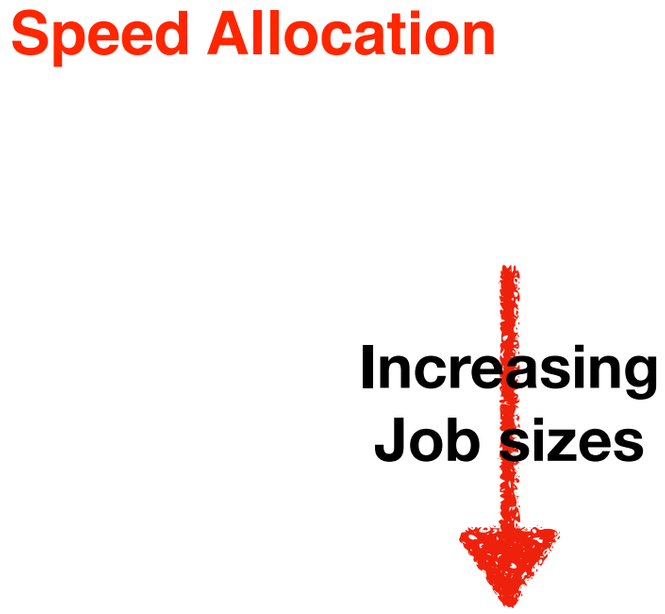
Prior Work





Multiple Servers



All jobs available at time 0, Optimal Scheduling : `heSRPT` **[Berg et al' 20]**

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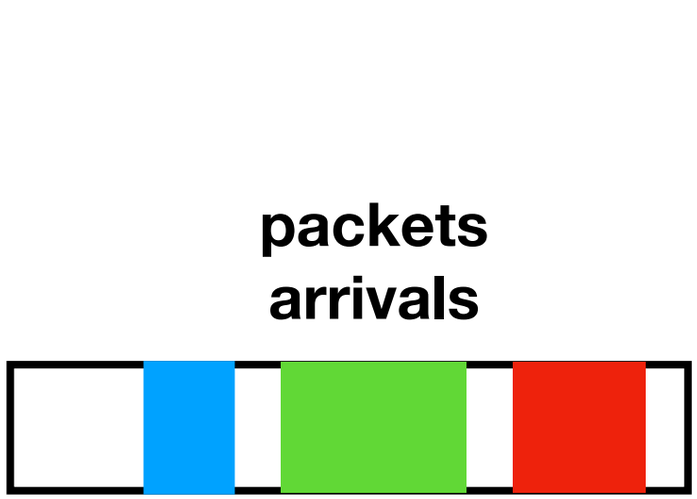
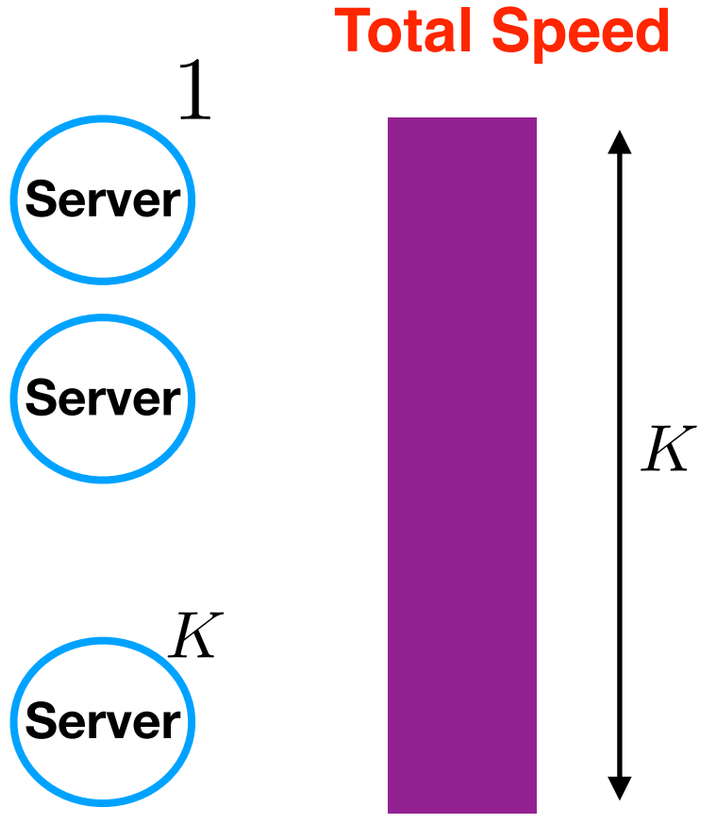
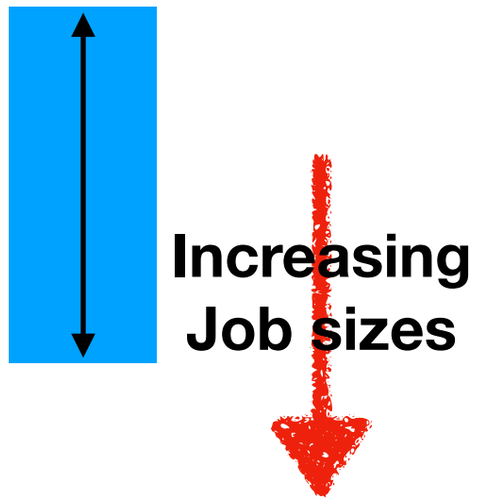
Prior Work





Multiple Servers



All jobs available at time 0, Optimal Scheduling : heSRPT **[Berg et al' 20]**

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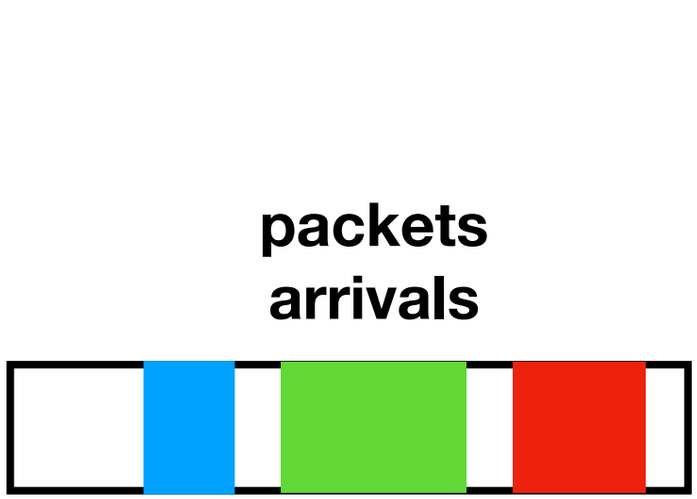
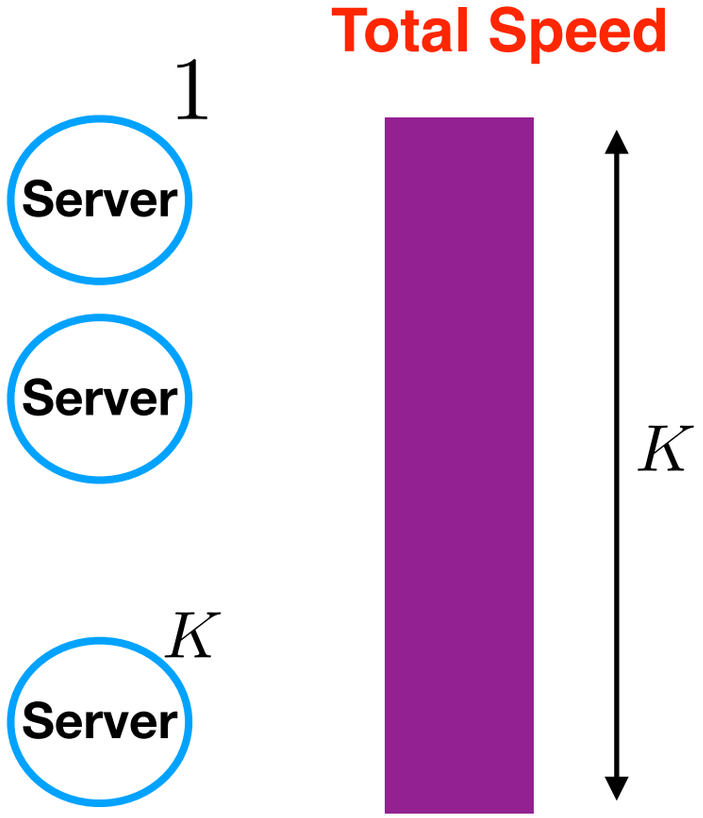
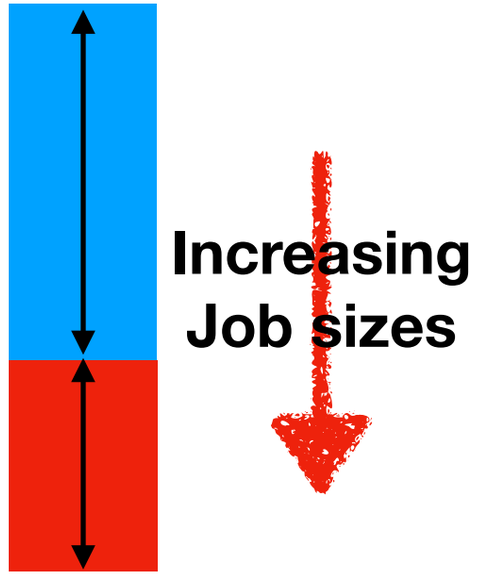
Prior Work





Multiple Servers



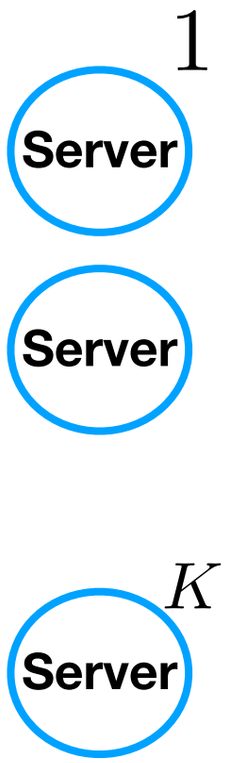
All jobs available at time 0, Optimal Scheduling : heSRPT **[Berg et al' 20]**

All jobs get non-zero speed, while shorter jobs get more speed

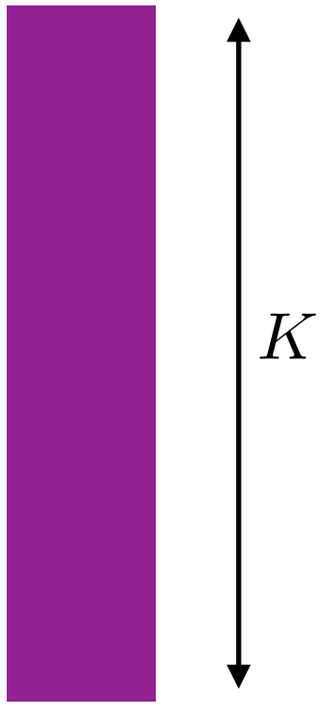
Prior Work

packets arrivals

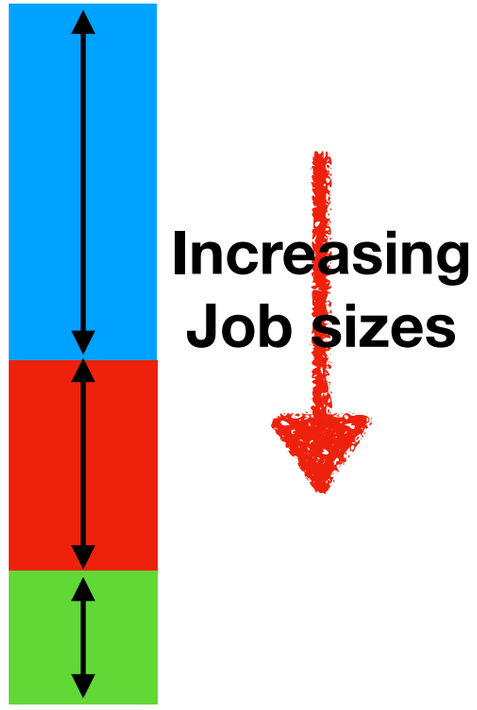
Multiple Servers



Total Speed



Speed Allocation



All jobs available at time 0, Optimal Scheduling : heSRPT **[Berg et al' 20]**

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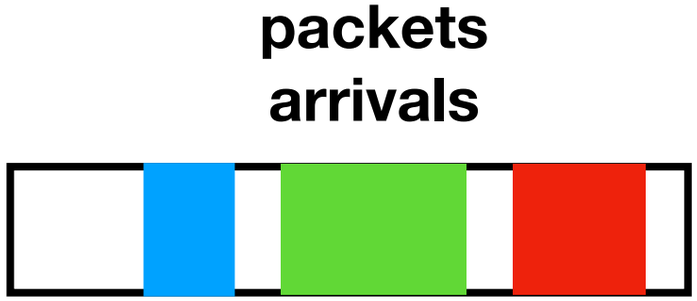
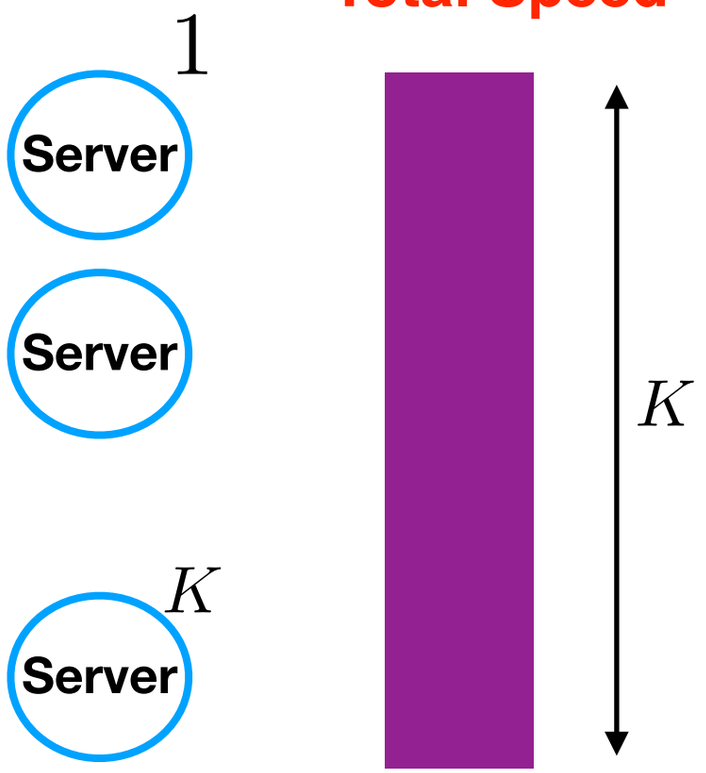
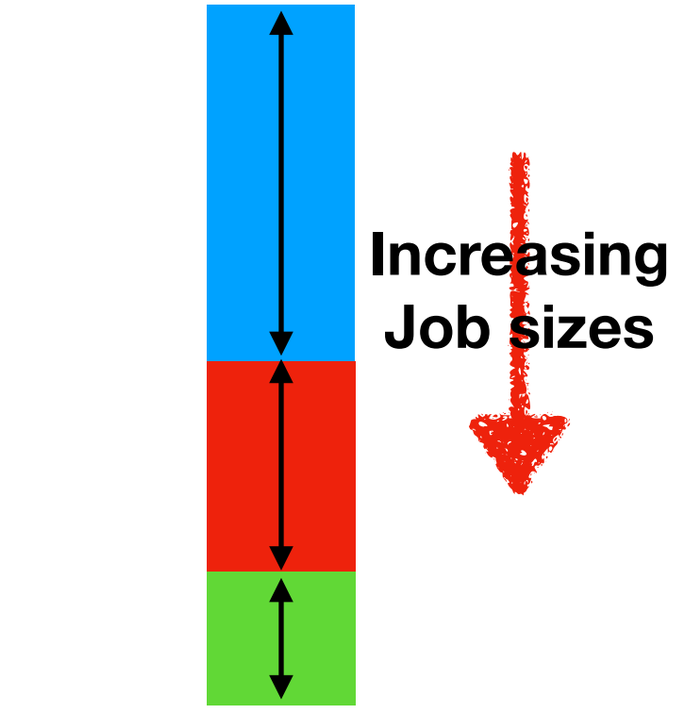
Prior Work





Multiple Servers



All jobs available at time 0, Optimal Scheduling : heSRPT **[Berg et al' 20]**

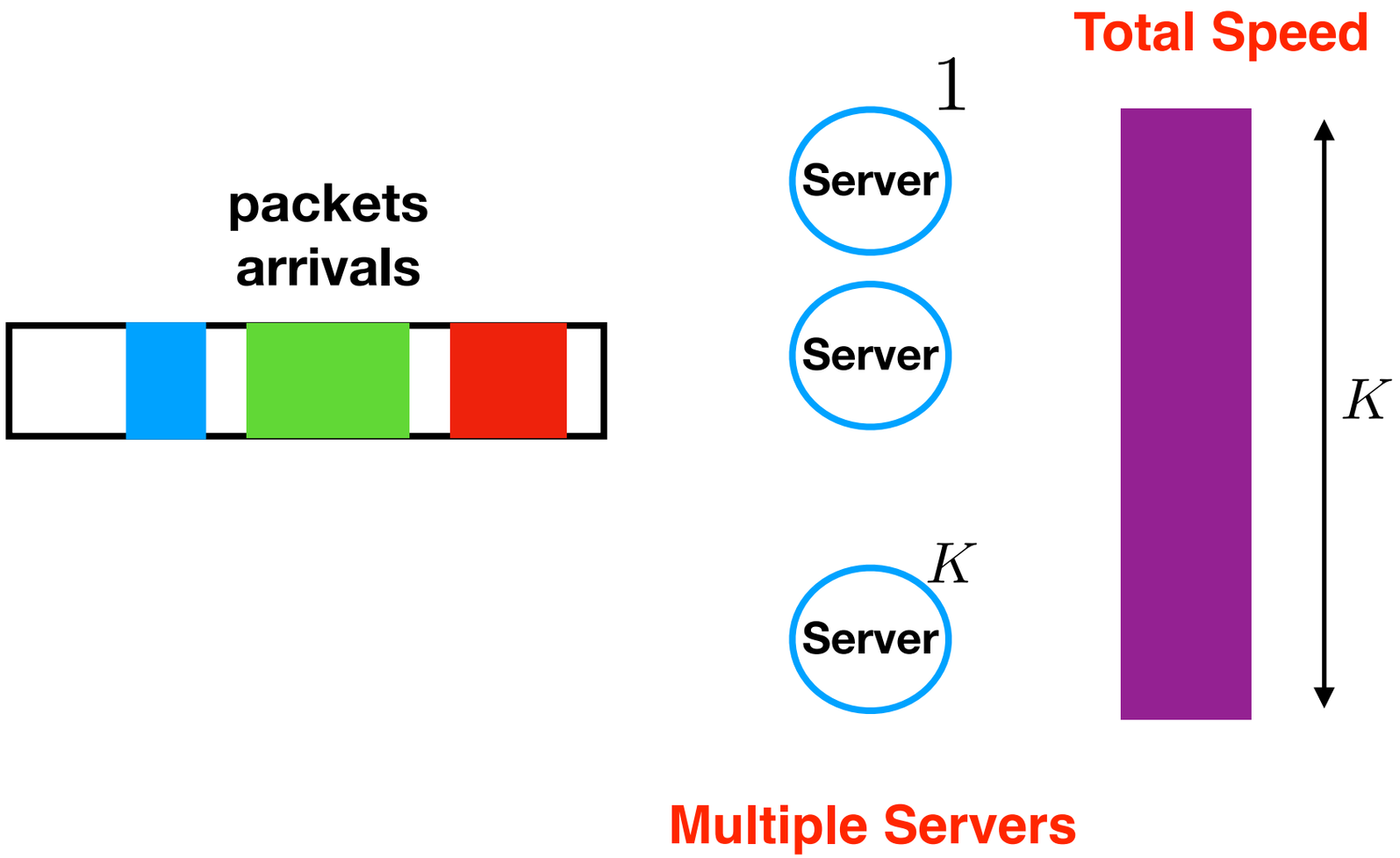
All jobs get non-zero speed, while shorter jobs get more speed

Among n(t) outstanding jobs

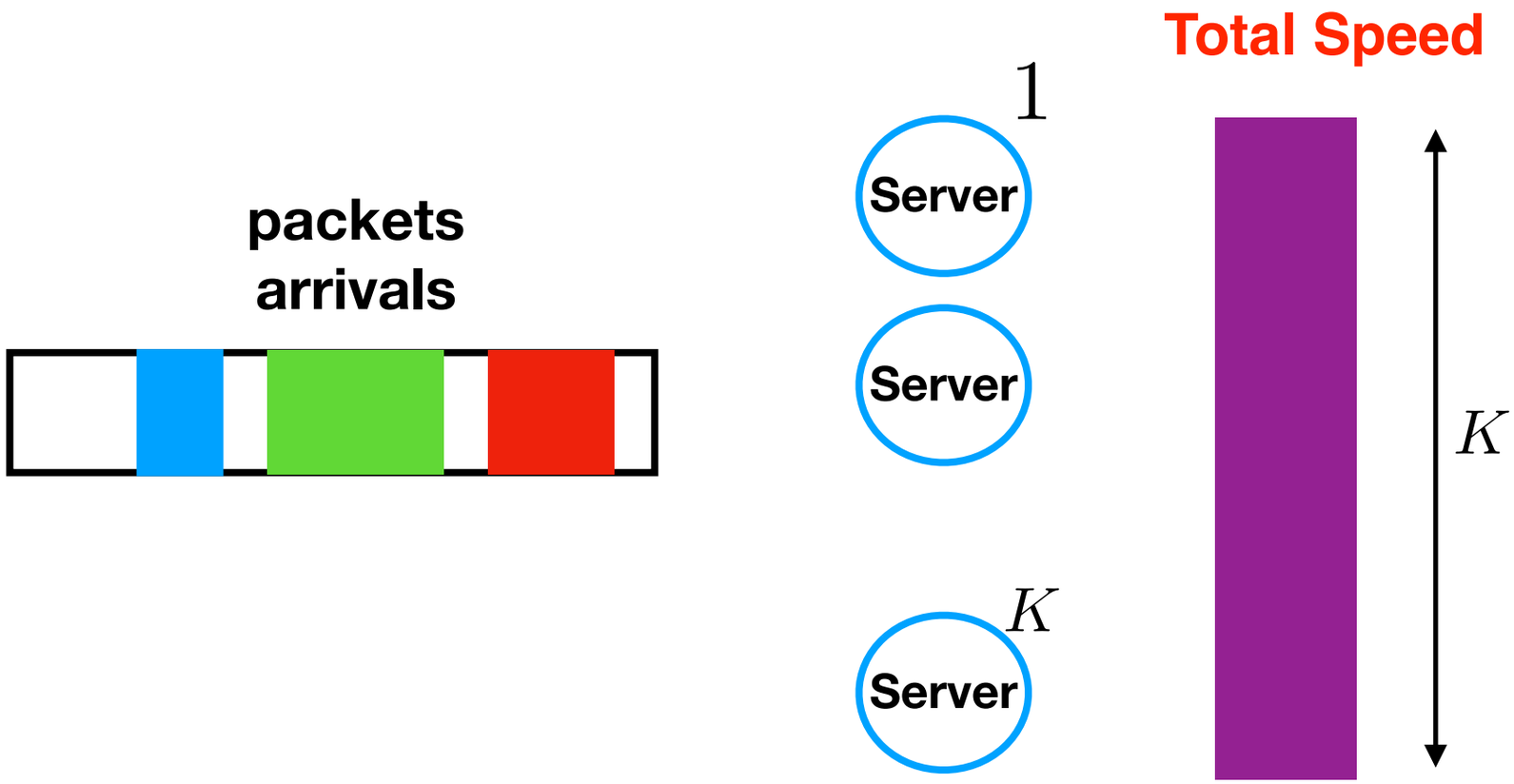
ith shortest job's speed

$$s_i(t) = K\left(\left(\frac{i}{n(t)}\right)^{\left(\frac{1}{1-1/\alpha}\right)} - \left(\frac{i-1}{n(t)}\right)^{\left(\frac{1}{1-1/\alpha}\right)}\right)$$

Simple Algorithm EQUI



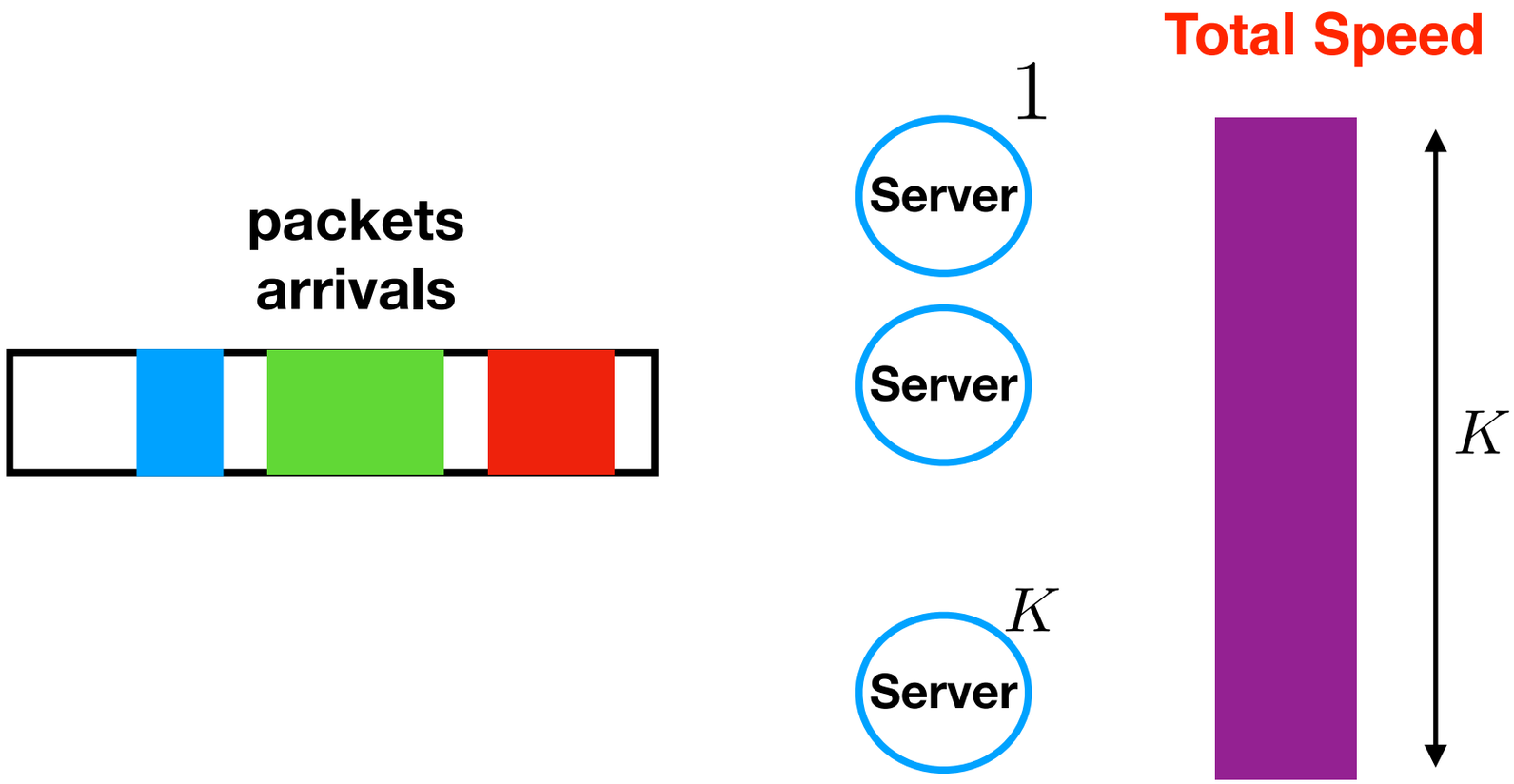
Simple Algorithm EQUI



Multiple Servers

All jobs available at time 0,

Simple Algorithm EQUI

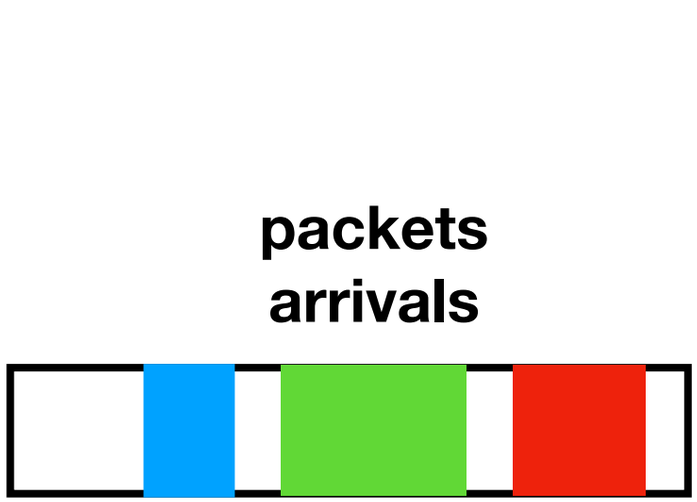


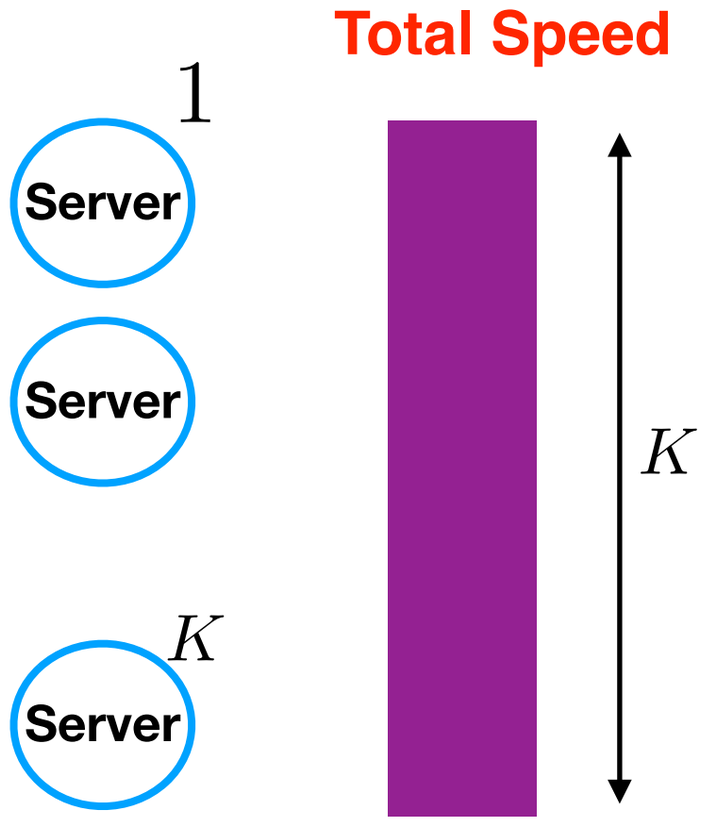
Multiple Servers

All jobs available at time 0,

Assign equal speed to all jobs

Simple Algorithm EQUI



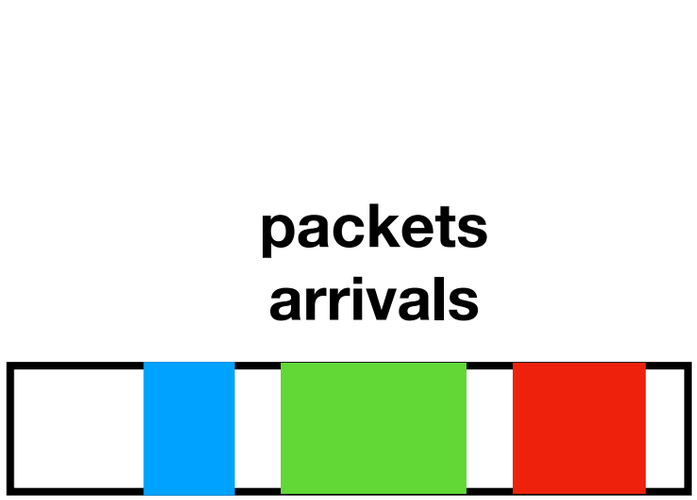


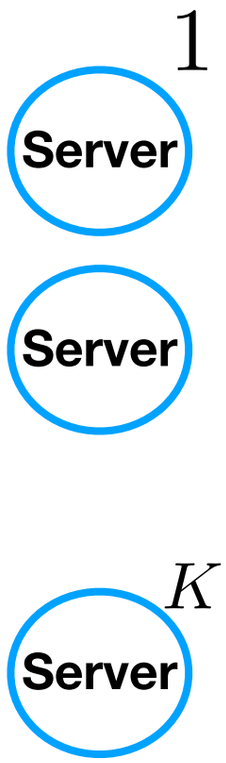
Multiple Servers

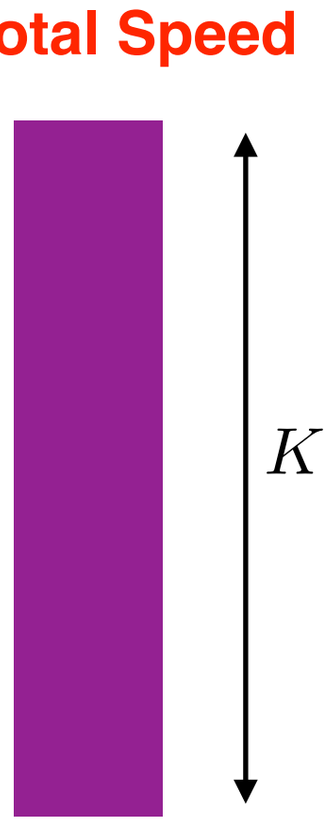
All jobs available at time 0,

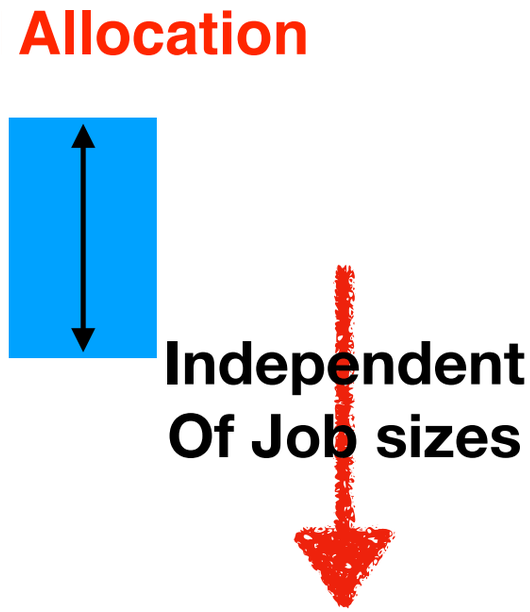
Assign equal speed to all jobs

Simple Algorithm EQUI







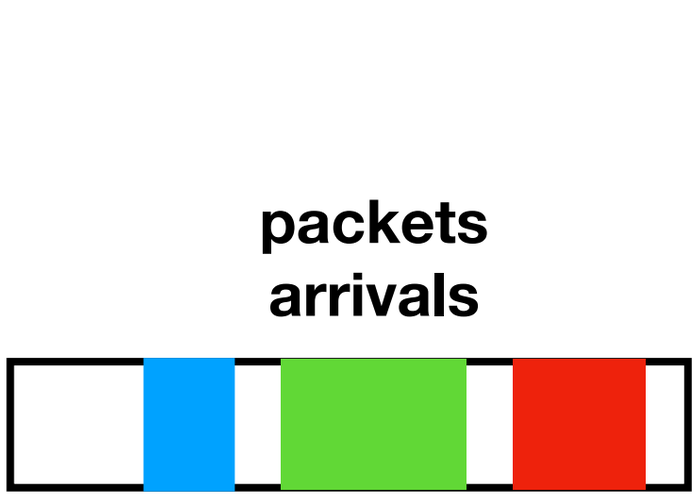


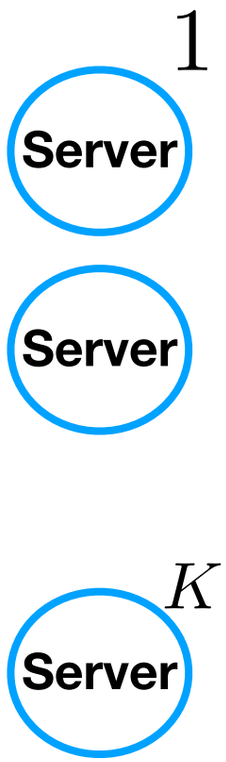
Multiple Servers

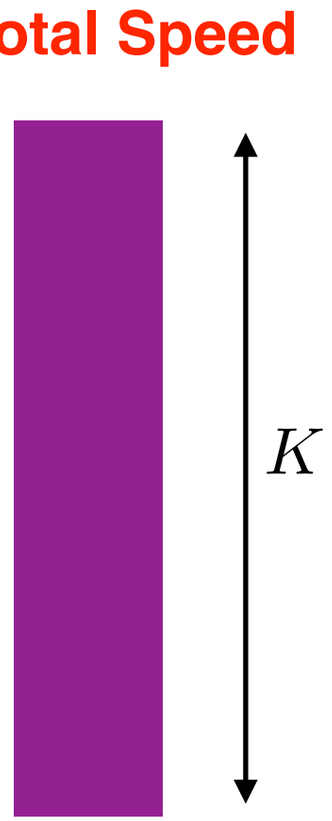
All jobs available at time 0,

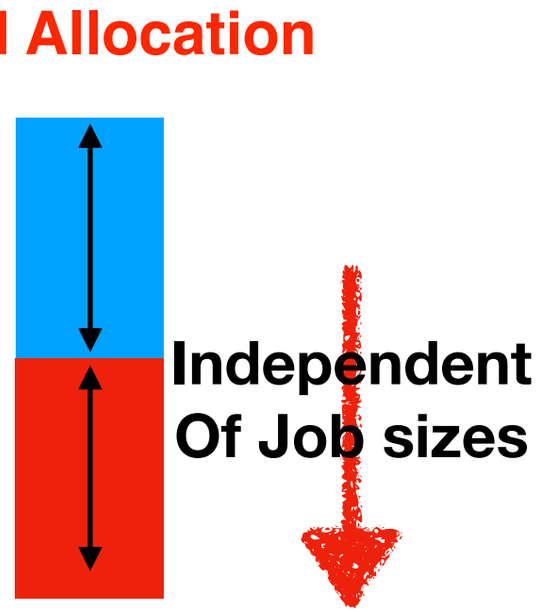
Assign equal speed to all jobs

Simple Algorithm EQUI







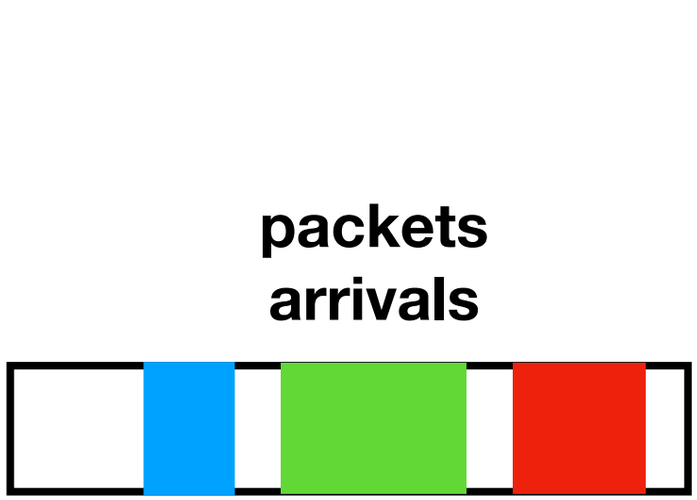


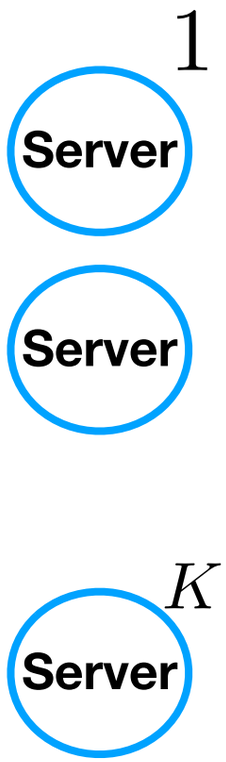
Multiple Servers

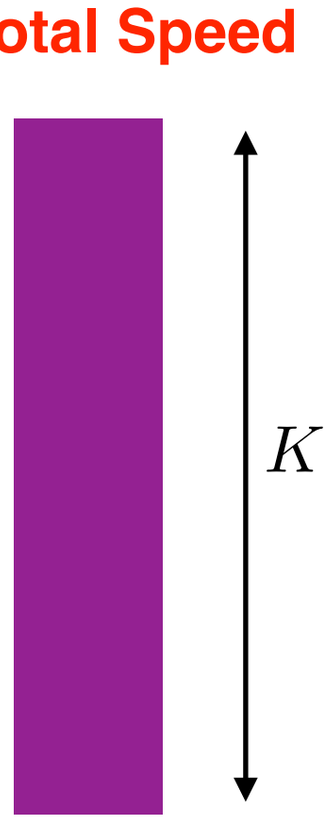
All jobs available at time 0,

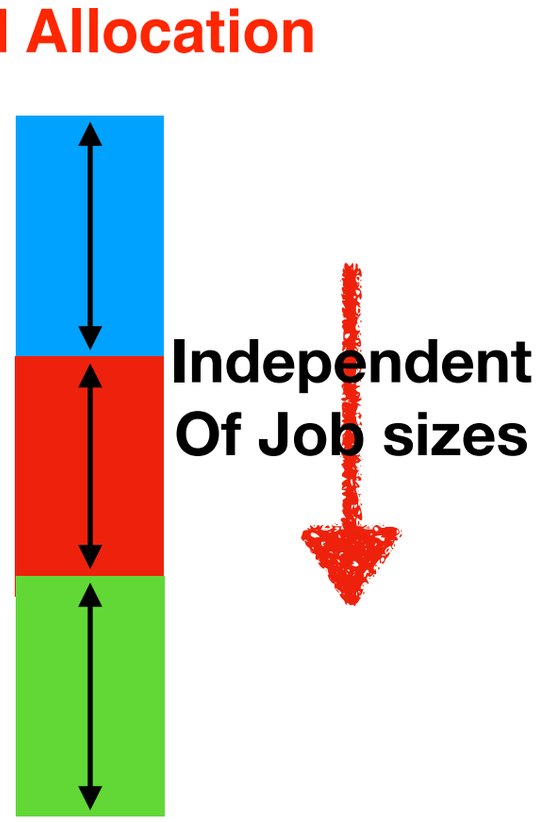
Assign equal speed to all jobs

Simple Algorithm EQUI









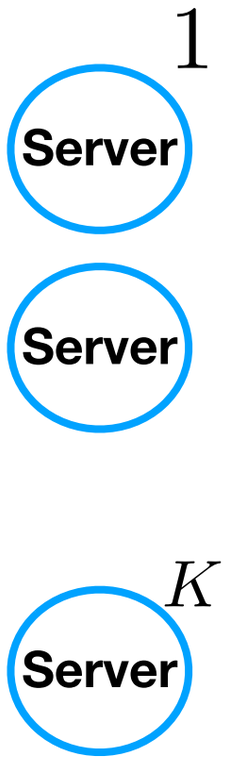
Multiple Servers

All jobs available at time 0,

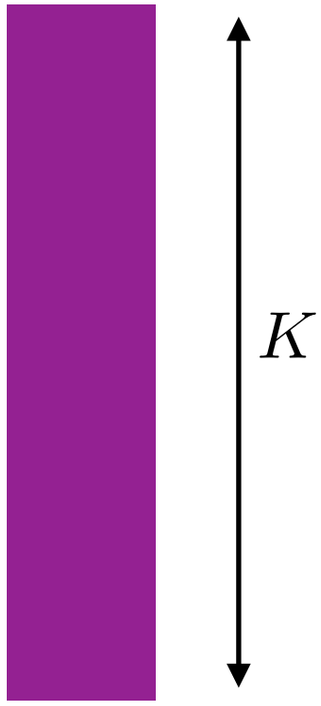
Assign equal speed to all jobs

Simple Algorithm EQUI

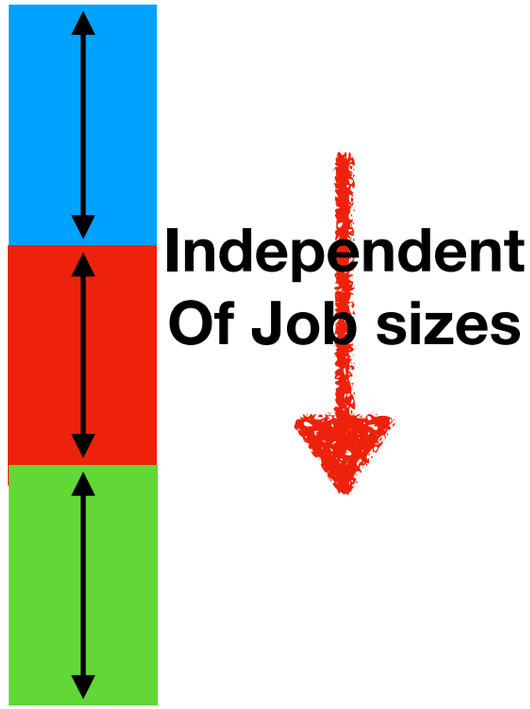
packets arrivals



Total Speed

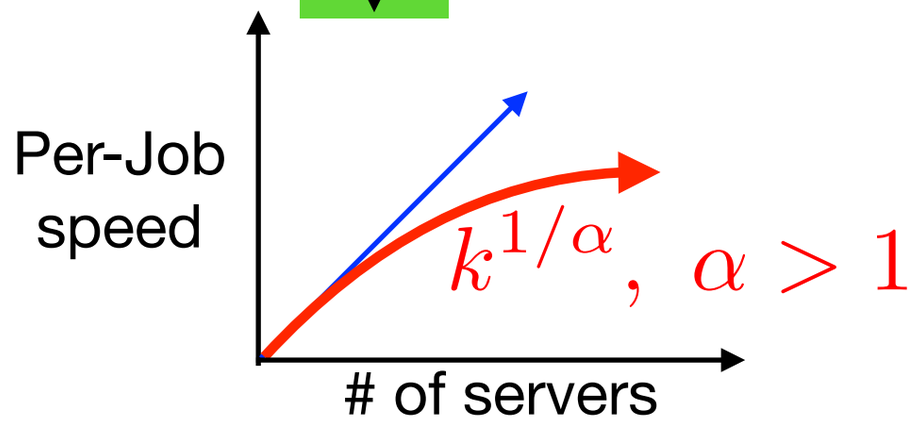


Speed Allocation



Multiple Servers

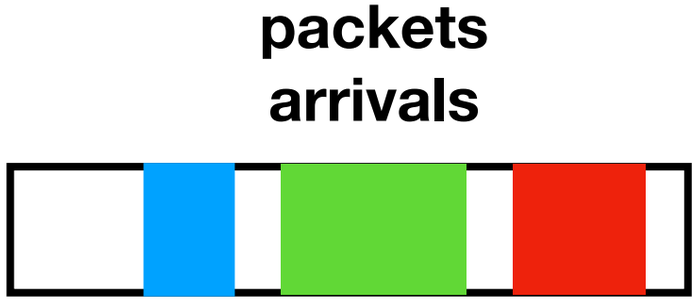
All jobs available at time 0,

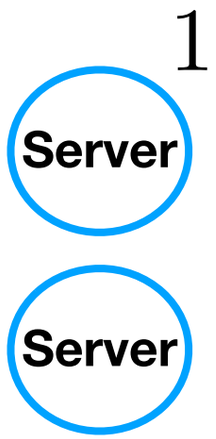


Assign equal speed to all jobs

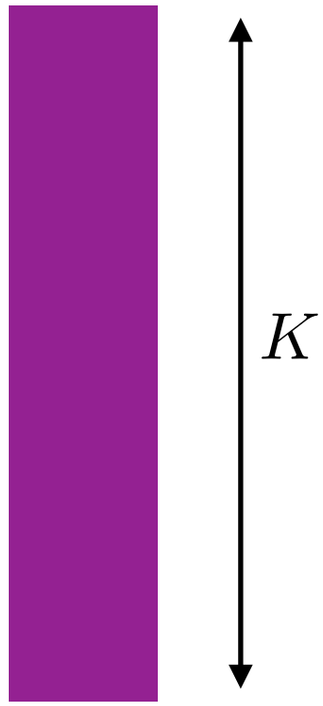
We show that Approximation Ratio of EQUI $\left(2 - \frac{1}{\alpha}\right) \frac{1}{\left(1 - \left(\frac{1}{\alpha}\right)\right)}$

Simple Algorithm EQUI

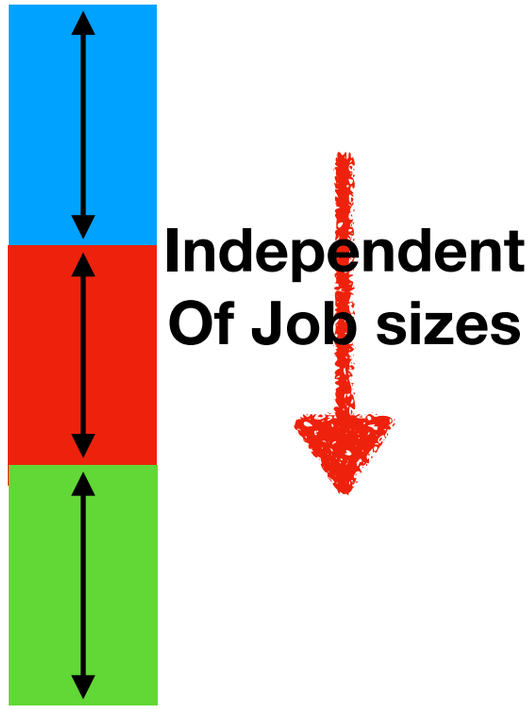




Total Speed

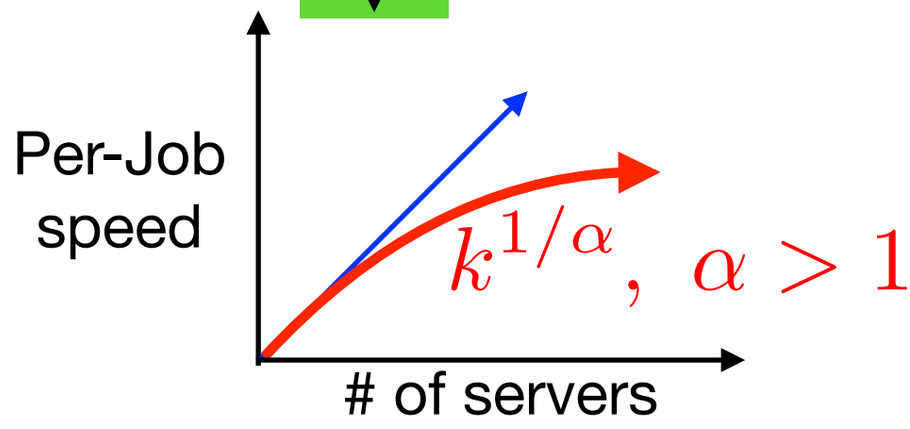


Speed Allocation



Multiple Servers

All jobs available at time 0,



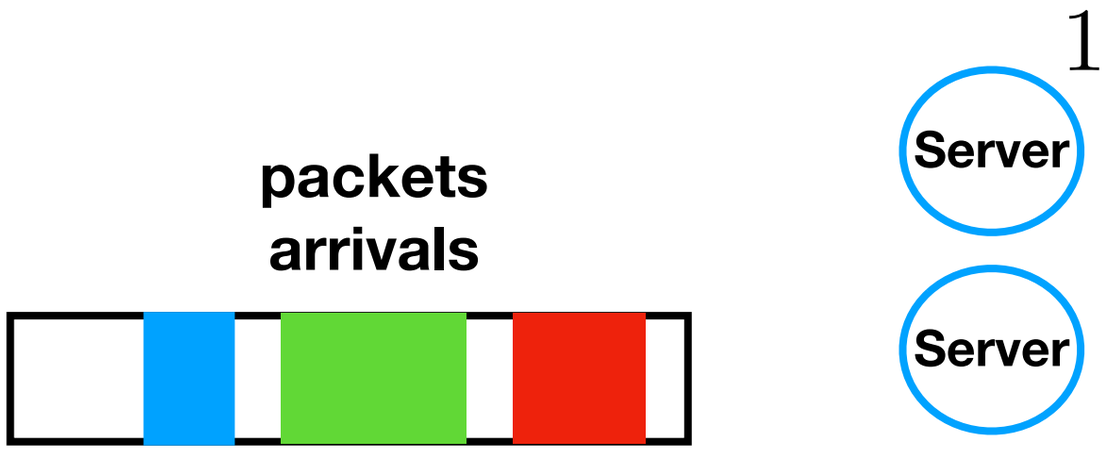
Assign equal speed to all jobs

We show that Approximation Ratio of EQUI $\left(2 - \frac{1}{\alpha}\right) \frac{1}{\left(1 - \left(\frac{1}{\alpha}\right)\right)}$

EQUI is simpler than heSRPT to implement

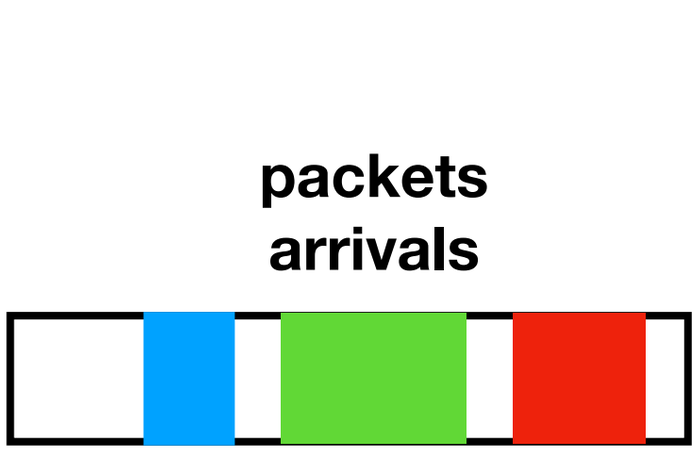
Real Problem: Online

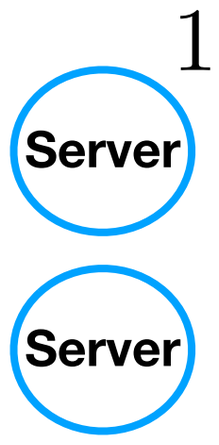
Real Problem: Online

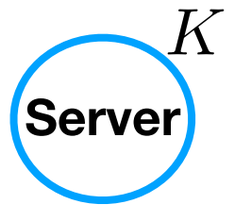


Multiple Servers

Real Problem: Online







Multiple Servers

Jobs arrive over time, arbitrary time and sizes

Metric for Online Algorithms

Metric for Online Algorithms

Competitive ratio

Metric for Online Algorithms

Competitive ratio ratio of the cost of an **online** and the **offline Opt** algorithm

$$r_{\mathsf{ON}} = \max_{\sigma} \frac{v_{\mathsf{ON}}(\sigma)}{v_{\mathsf{OPT}}(\sigma)}$$

Worst Case Input

Metric for Online Algorithms

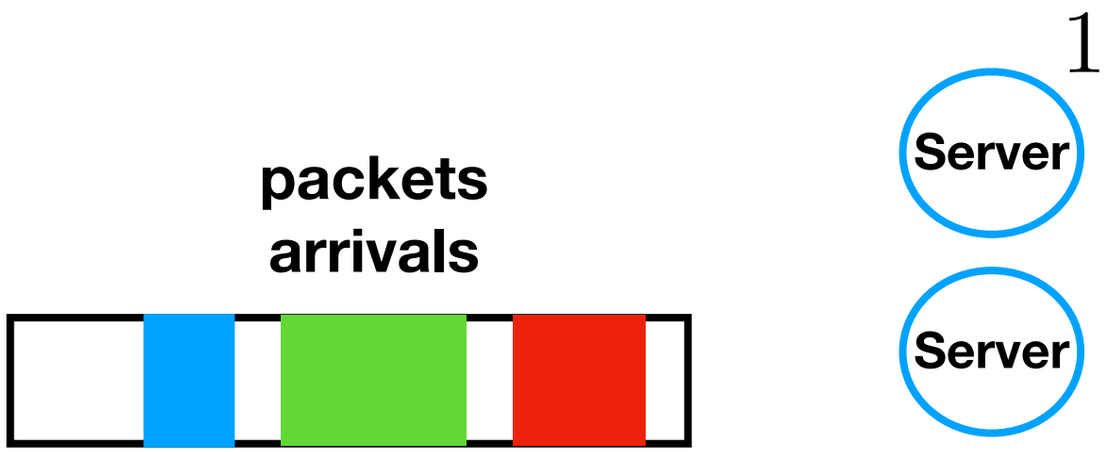
Competitive ratio ratio of the cost of an **online** and the **offline Opt** algorithm

$$r_{\mathsf{ON}} = \max_{\sigma} \frac{v_{\mathsf{ON}}(\sigma)}{v_{\mathsf{OPT}}(\sigma)}$$

Worst Case Input

Goal online algorithm with least **CR**

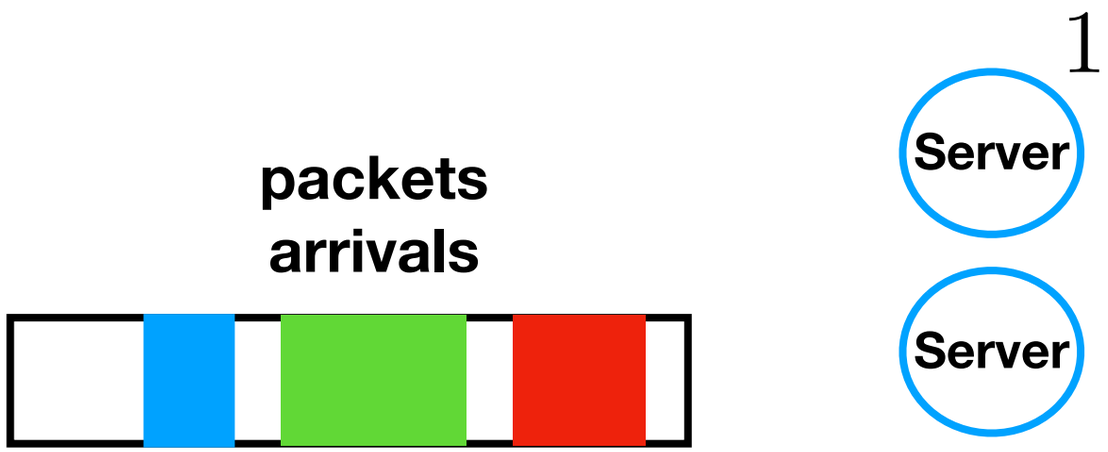
Prior Work : Online Case



Multiple Servers

Jobs arrive over time, arbitrary time and sizes

Prior Work : Online Case

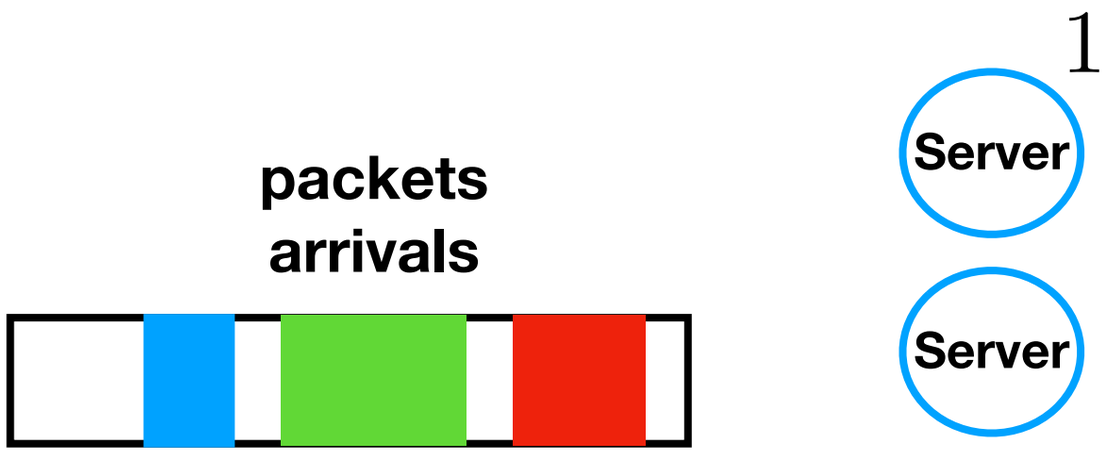


Multiple Servers

Jobs arrive over time, arbitrary time and sizes

Not much is known, unless resource augmentation is provided

Prior Work : Online Case



Multiple Servers

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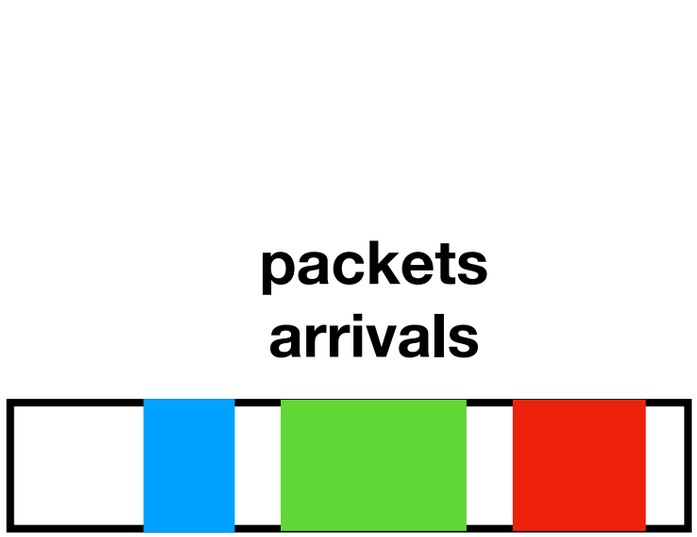
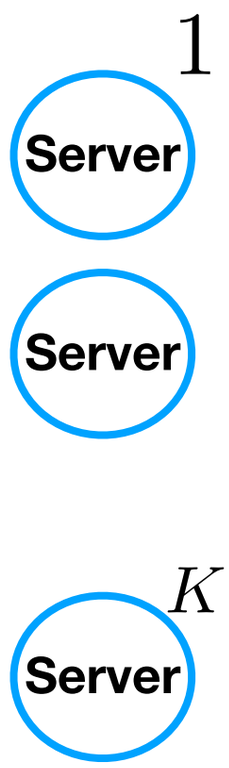
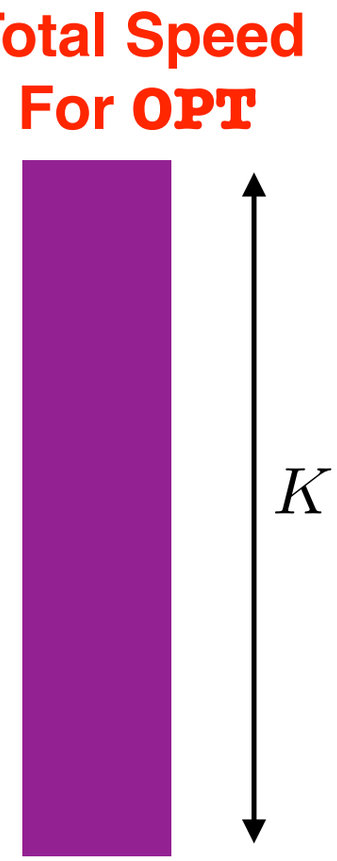
With resource augmentation an algorithm has more resources than OPT

Prior Work : Online Case







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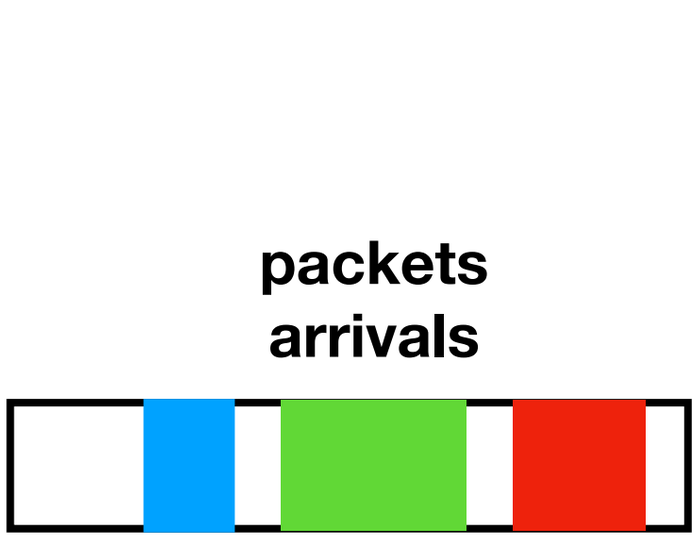
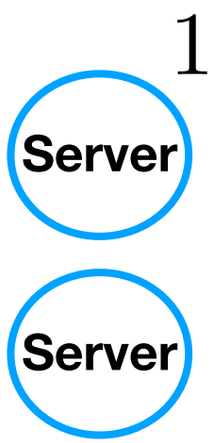
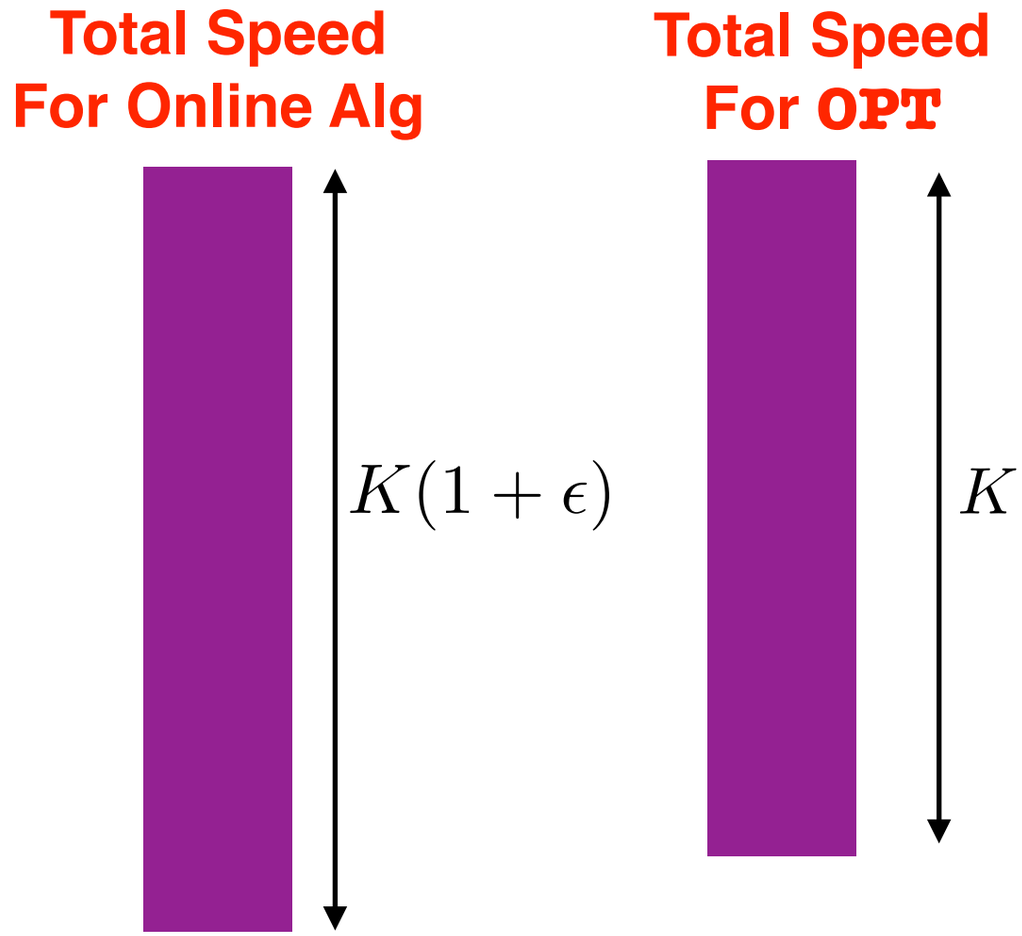




Multiple Servers



Jobs arrive over time, arbitrary time and sizes

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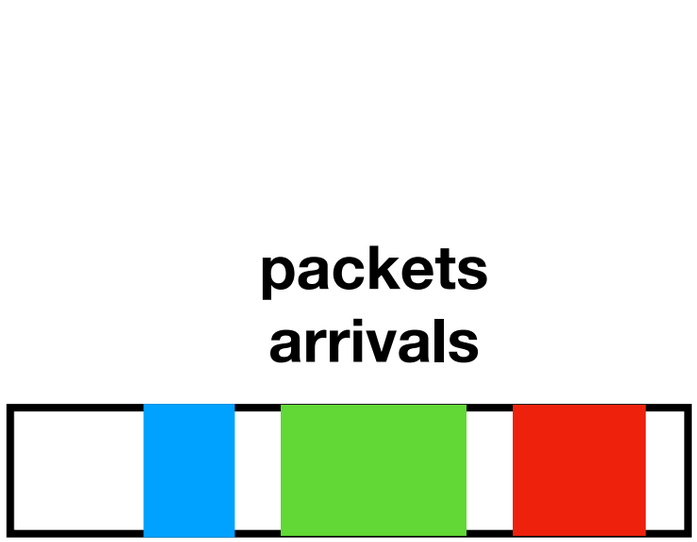
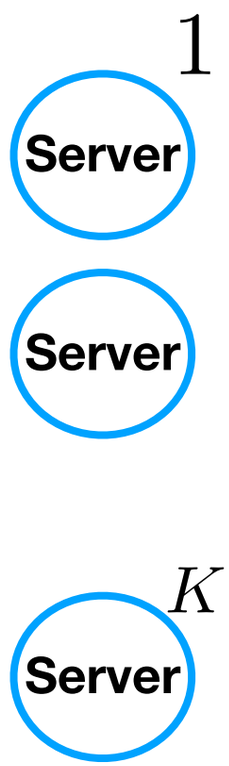
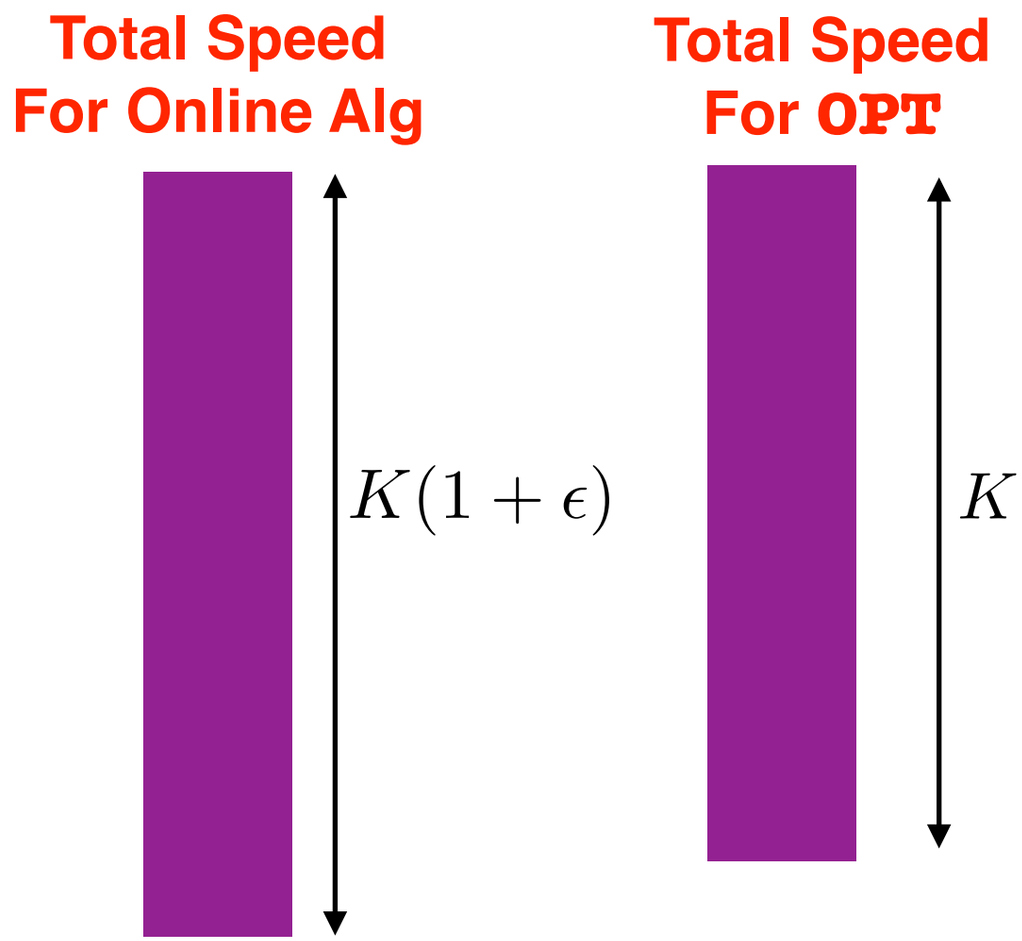
Prior Work : Online Case





Multiple Servers



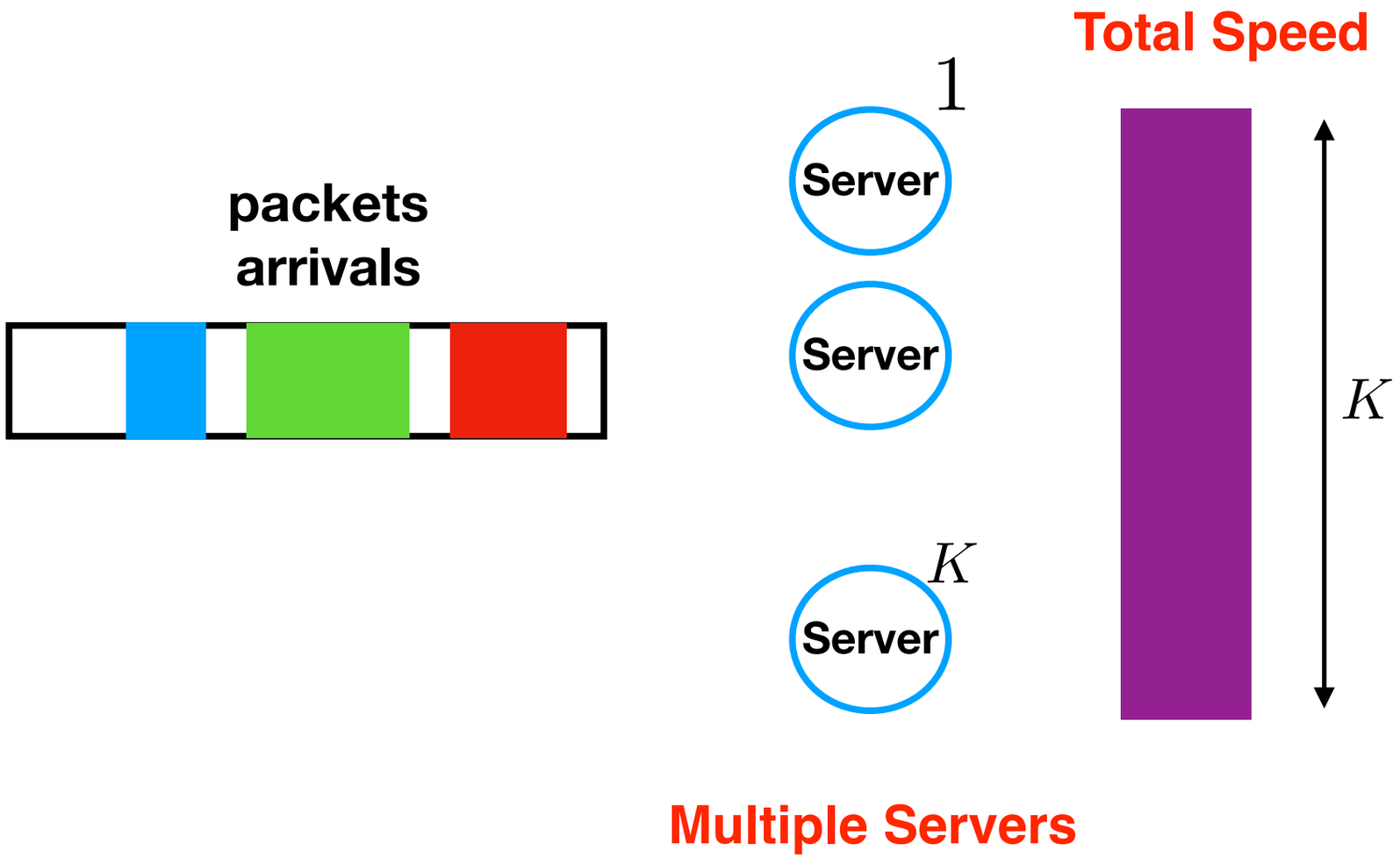
Jobs arrive over time, arbitrary time and sizes

Not much is known, unless resource augmentation is provided

With resource augmentation an algorithm has more resources than OPT

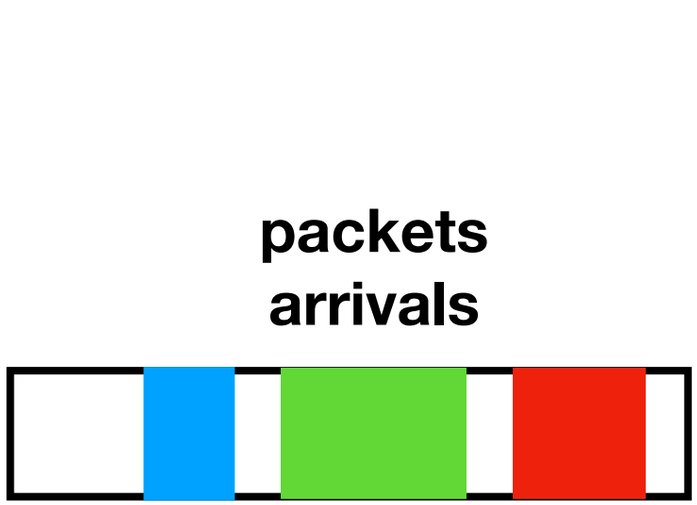
With resource augmentation constant competitive algorithms are known [*Edmonds* et al]

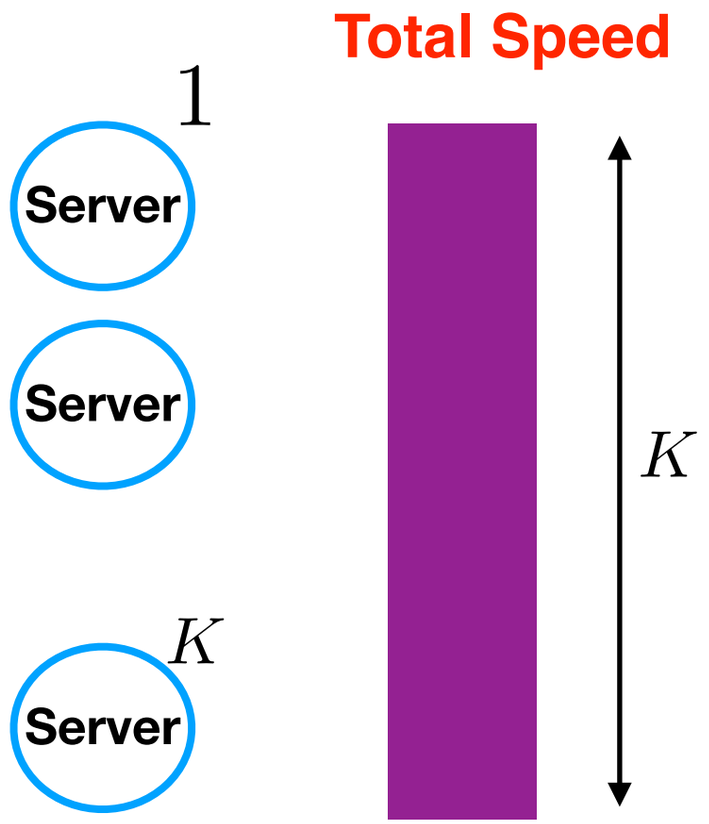
Our Algorithm



Multiple Servers

Our Algorithm LCFS-EQUI





Multiple Servers

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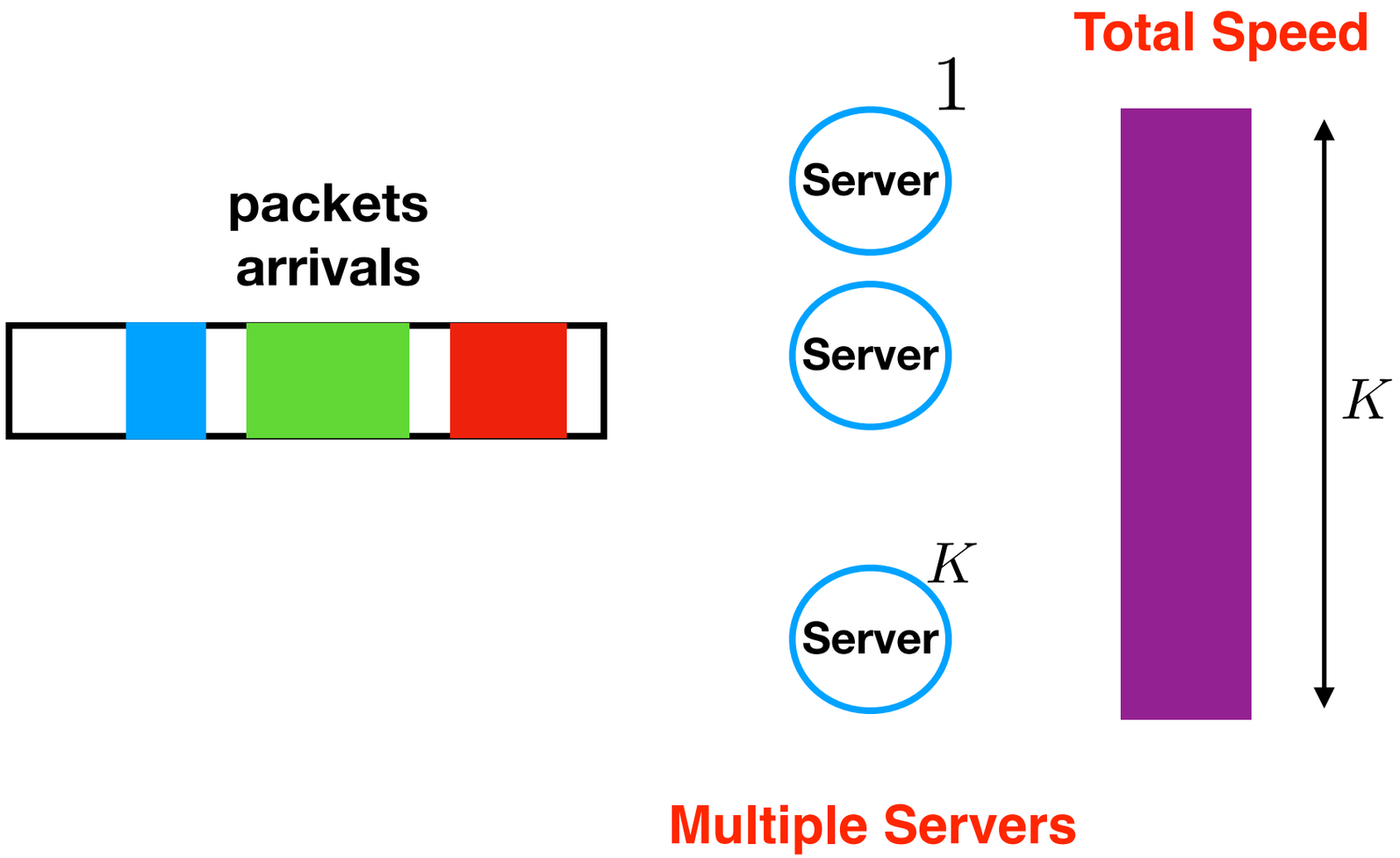
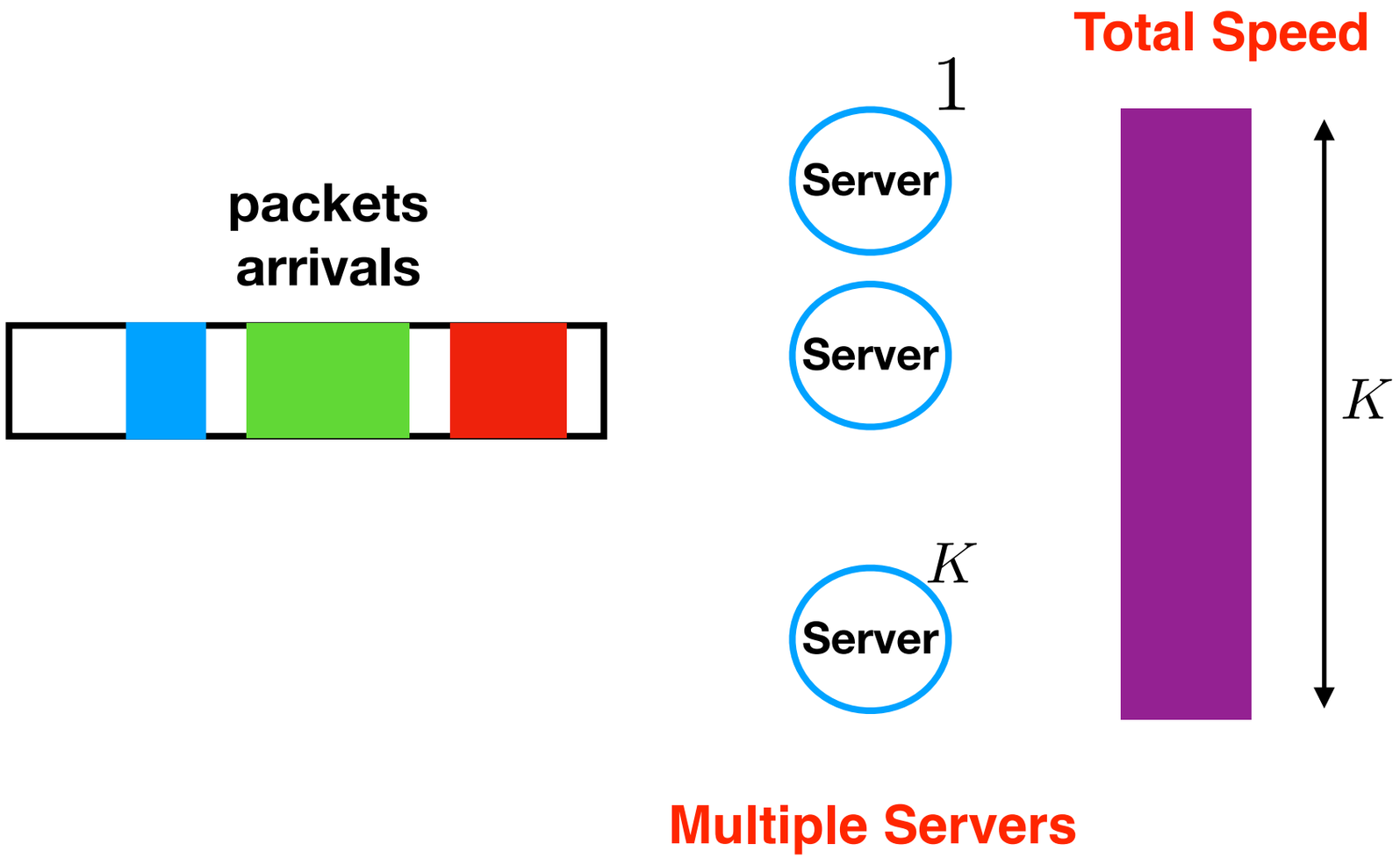


Multiple Servers

Process the β-**fraction** of the **most recently arrived** jobs

Our Algorithm LCFS-EQUI

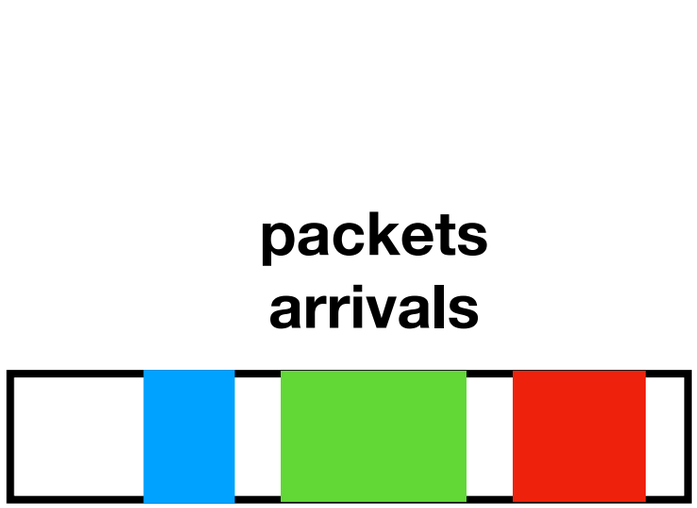


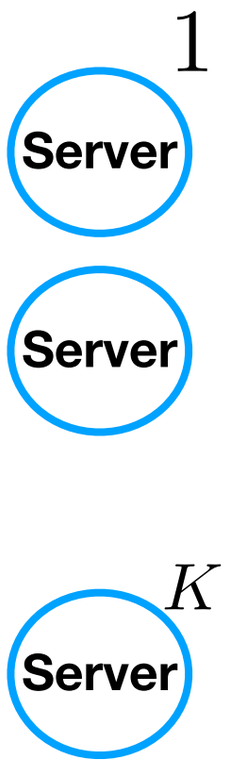
Multiple Servers

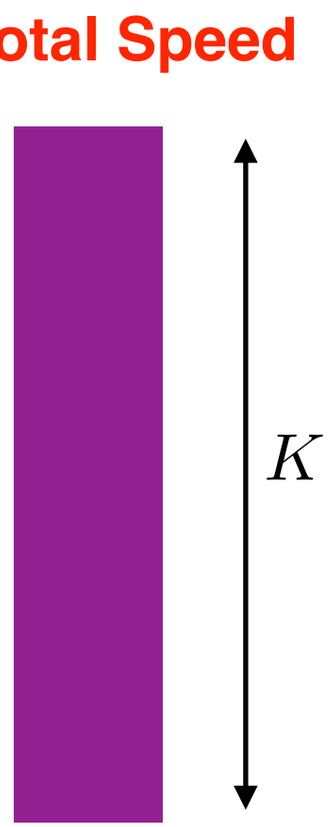
Process the β-**fraction** of the **most recently arrived** jobs

Assign **equal speed** to all jobs being processed

Our Algorithm LCFS-EQUI







Multiple Servers

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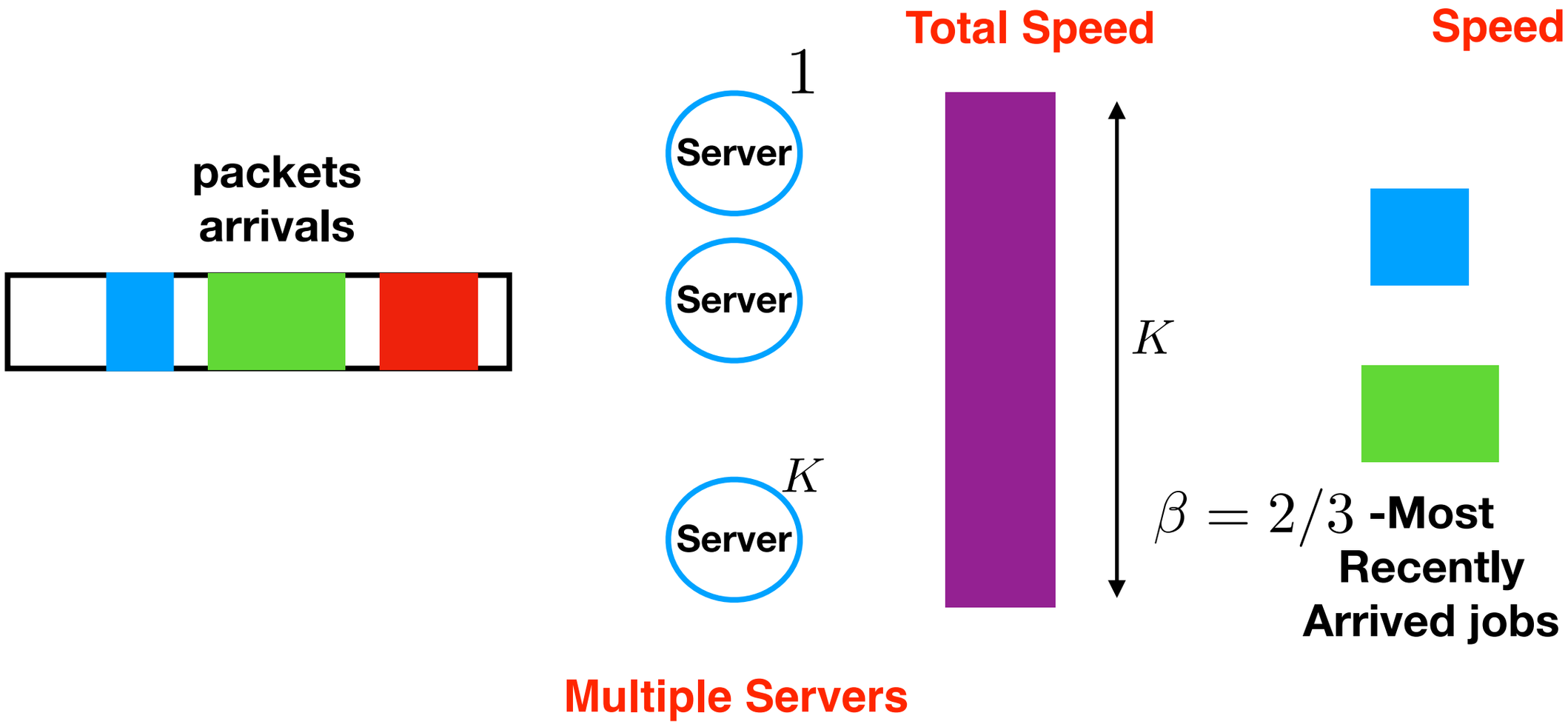

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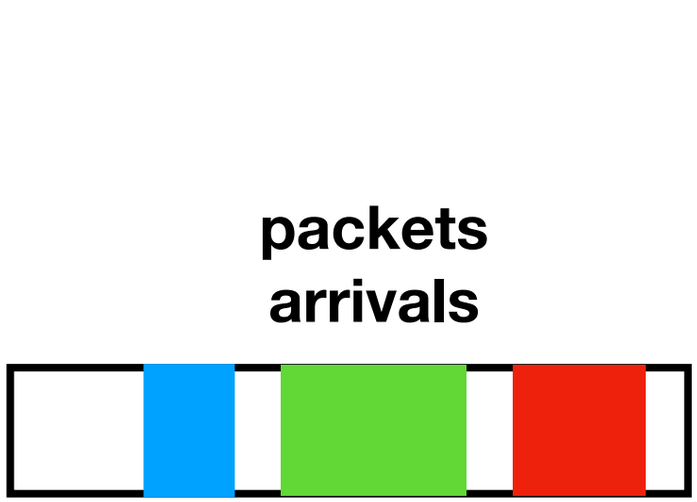

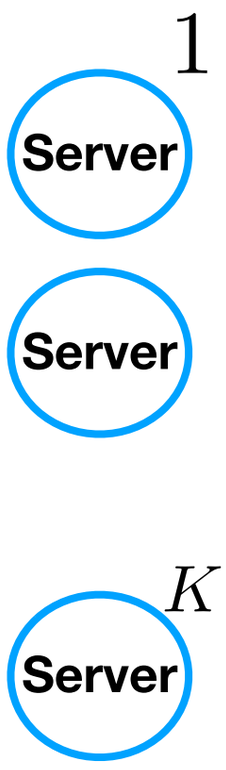

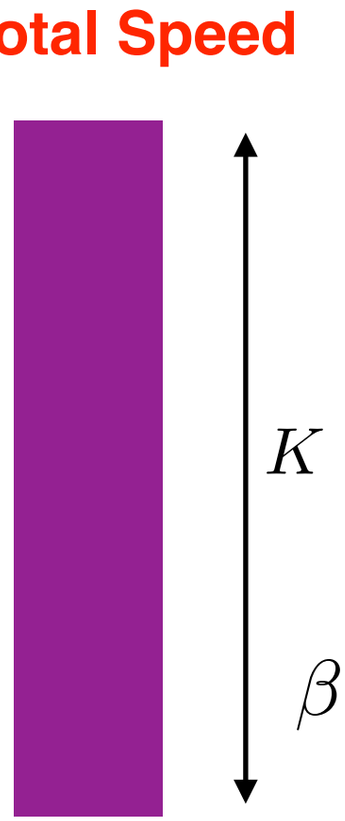

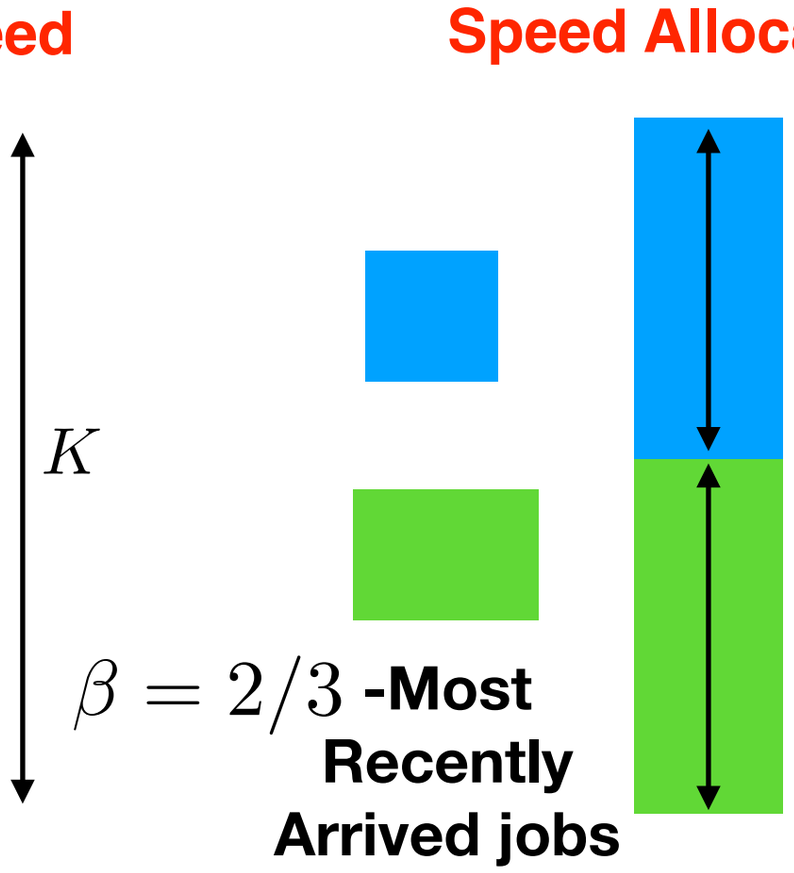

Multiple Servers

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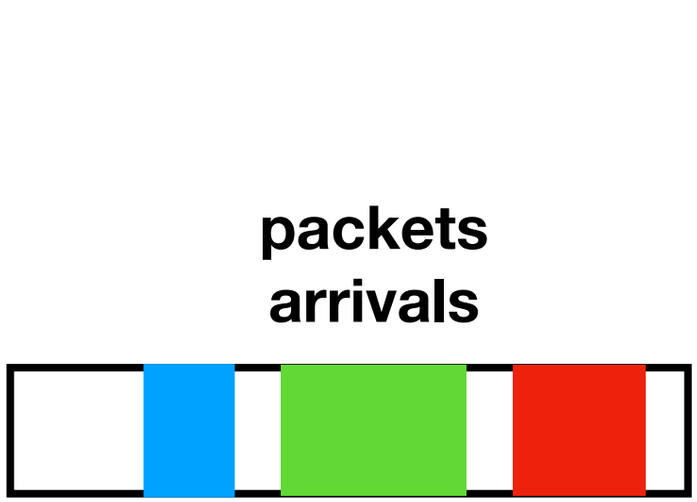
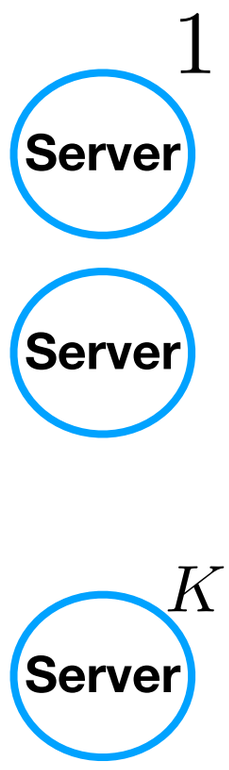
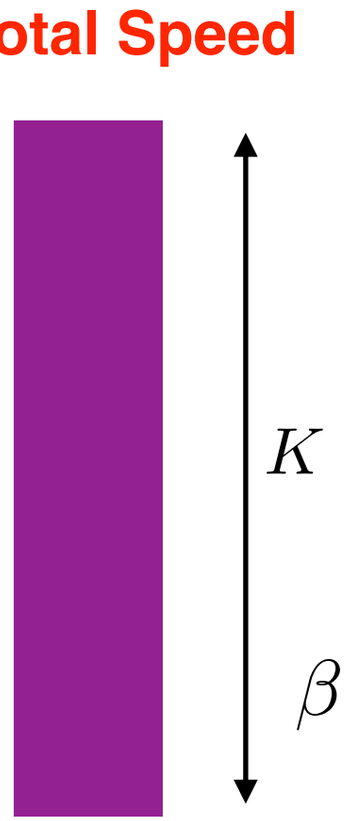
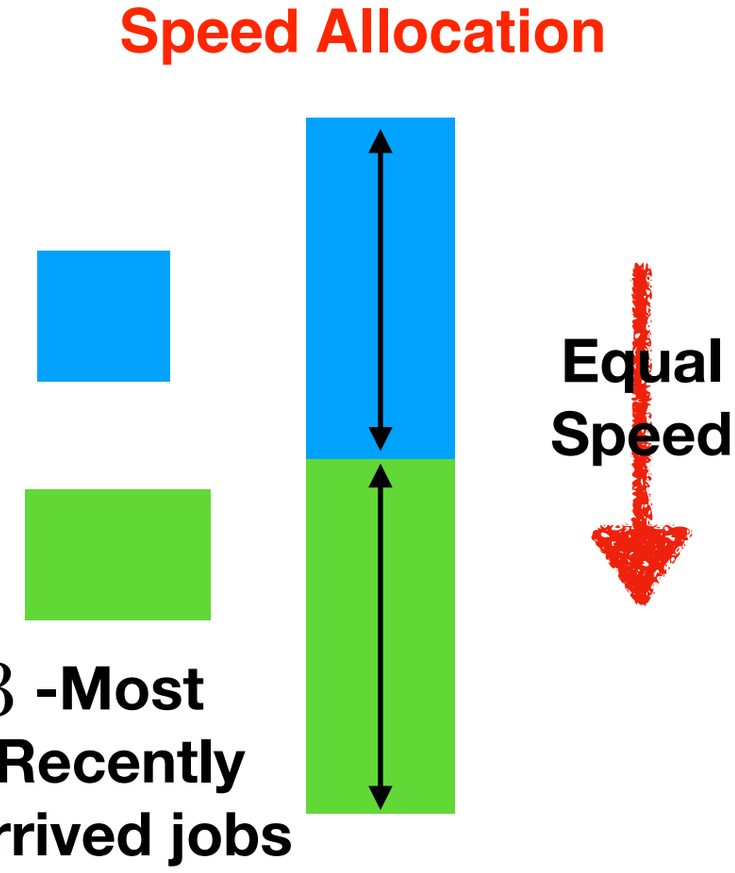




Multiple Servers



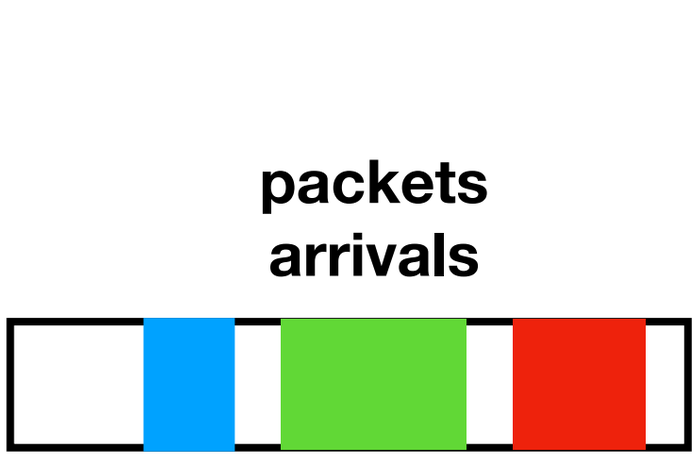


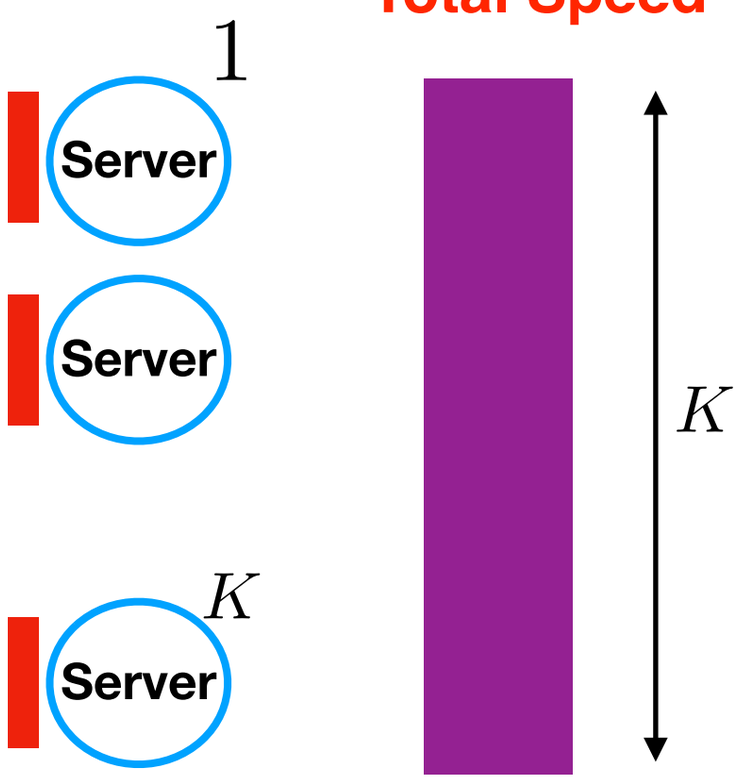
Process the β -**fraction** of the **most recently arrived** jobs

Assign **equal speed** to all jobs being processed

Algorithm is non-clairvoyant : no information needed about remaining job size

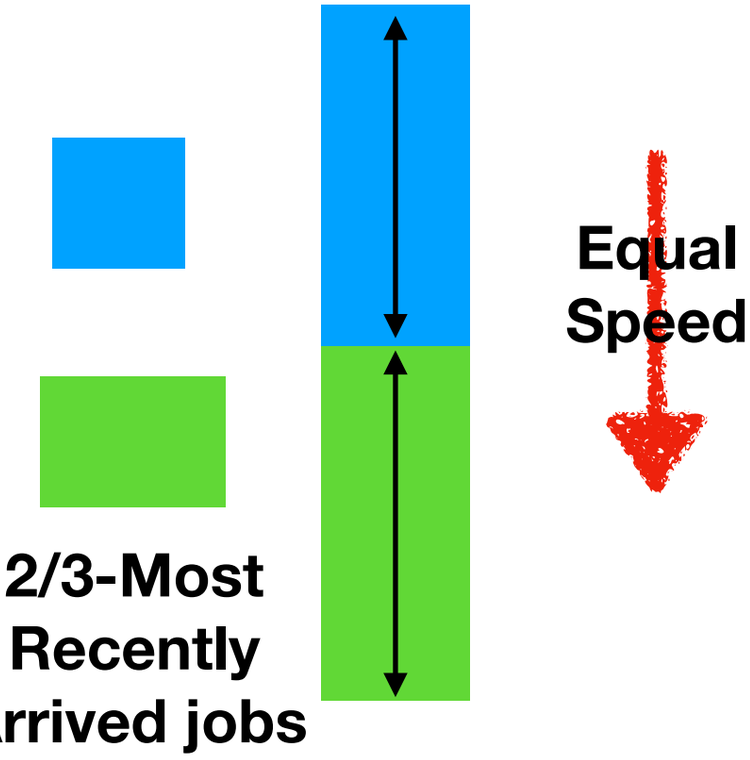
Intuition LCFS-EQUI





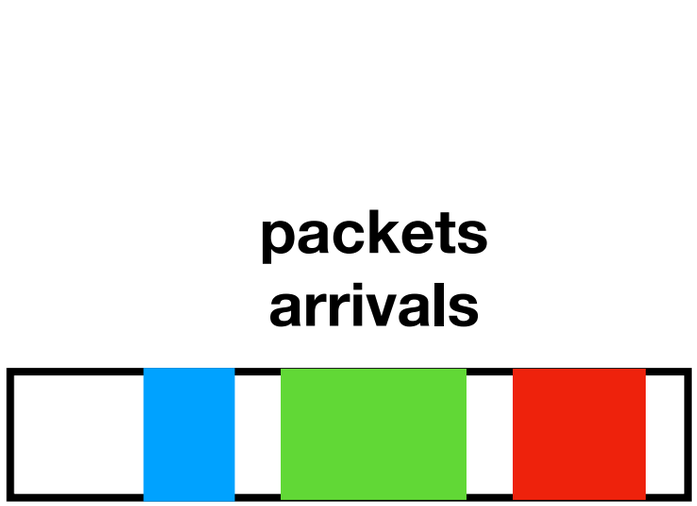
Multiple Servers

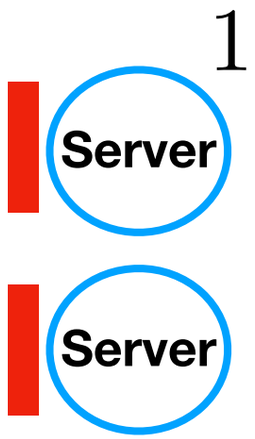
Total Speed

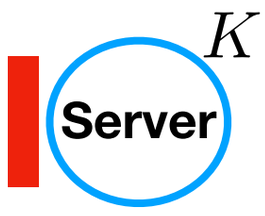


Speed Allocation

Intuition LCFS-EQUI







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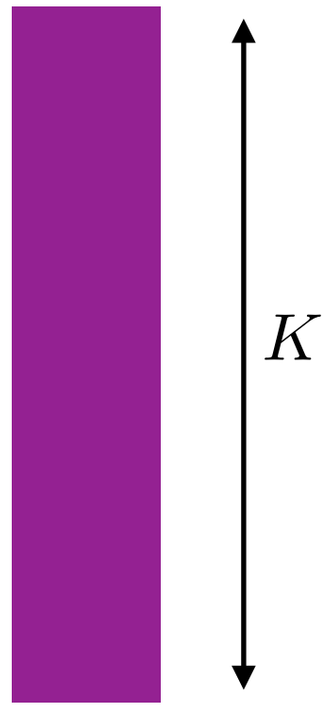
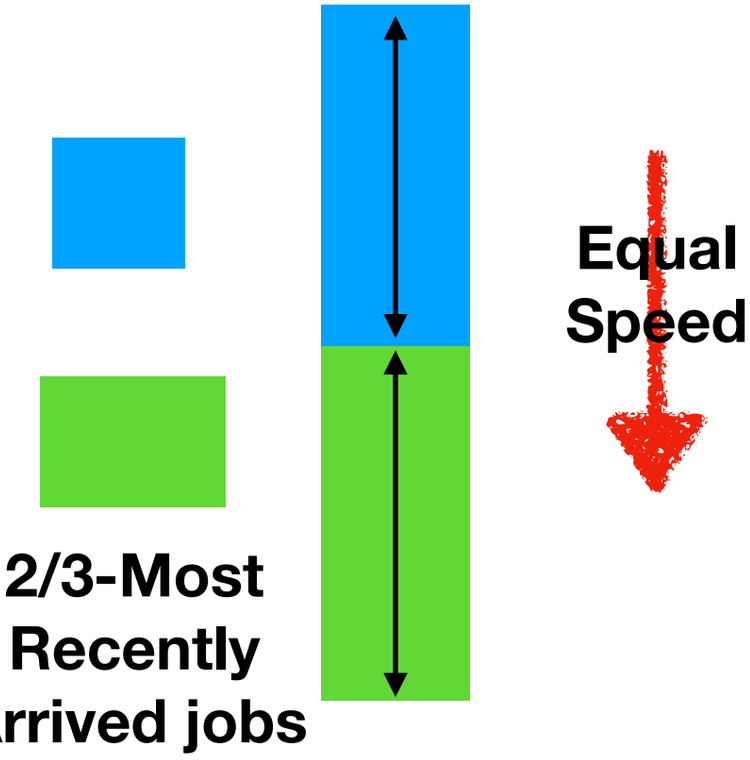
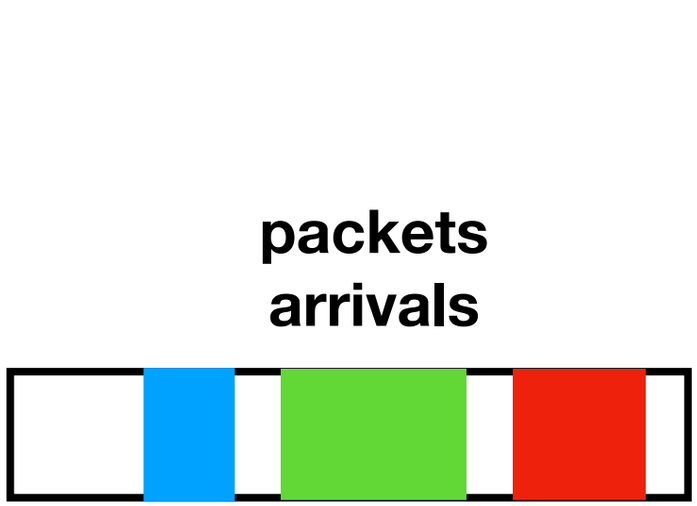
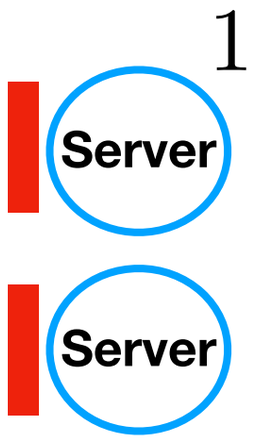


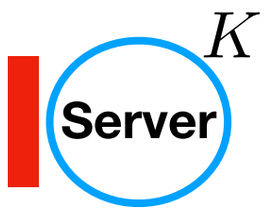


SRPT- short jobs remain in system for short time

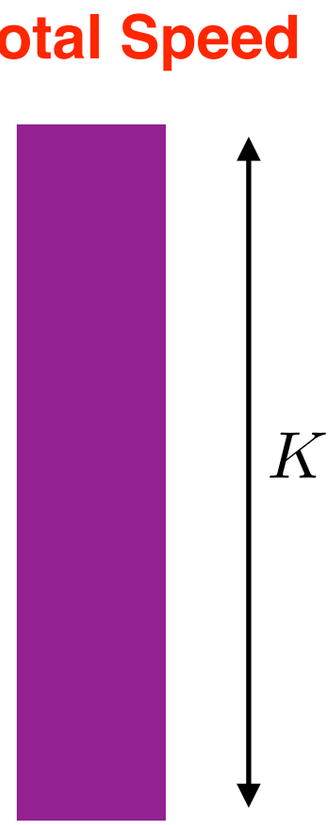
Intuition LCFS-EQUI

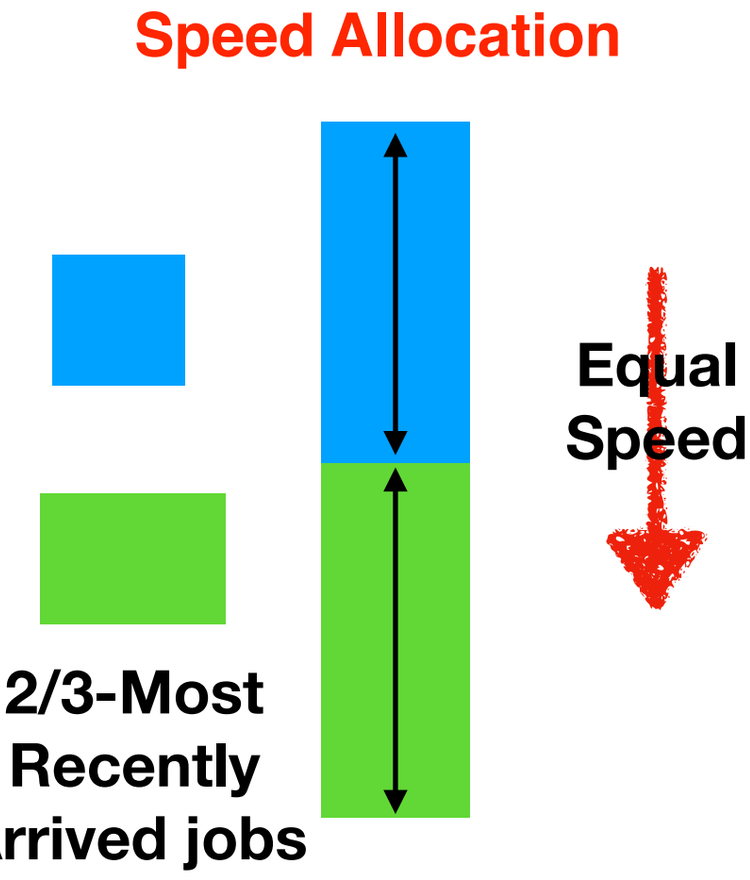






Multiple Servers

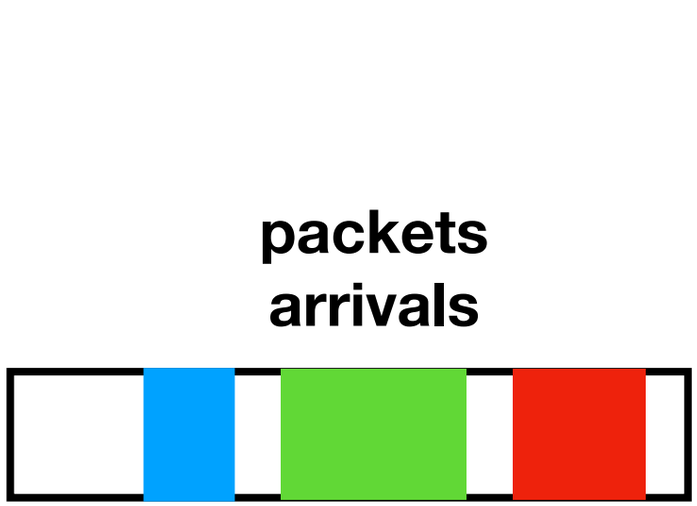


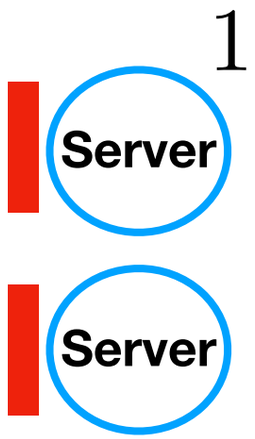


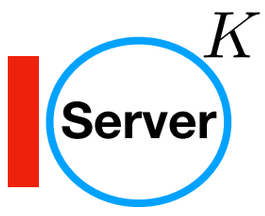
SRPT- short jobs remain in system for short time

LCFS- w/o job size information, long jobs remain in system for long

Intuition LCFS-EQUI

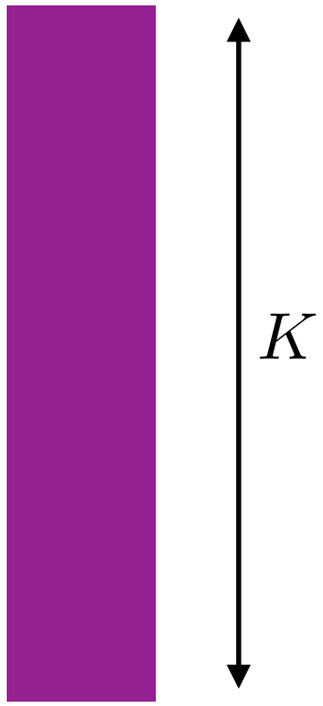




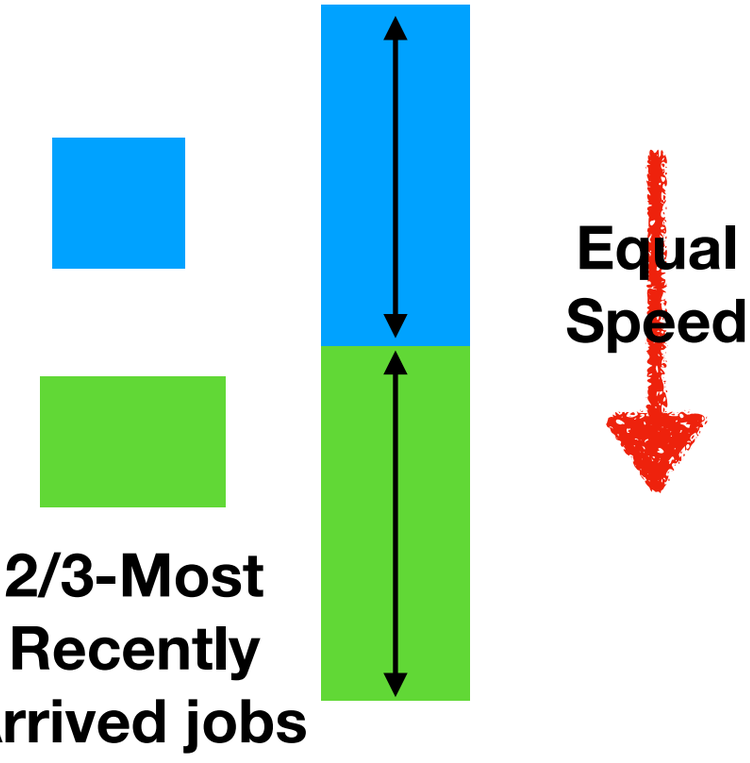


Multiple Servers

Total Speed



Speed Allocation

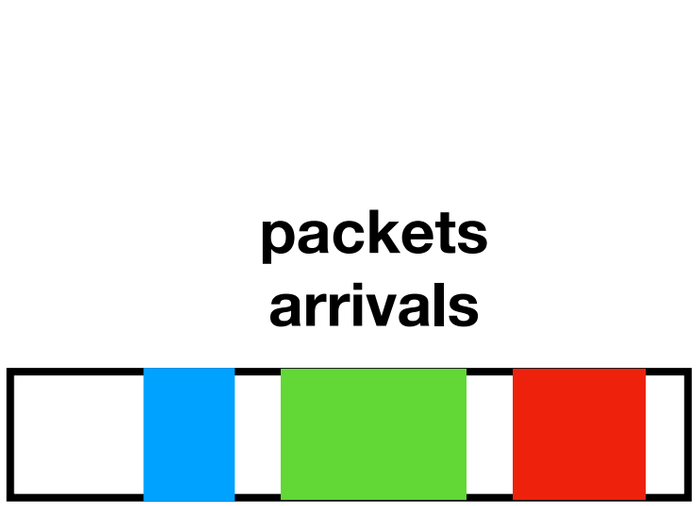


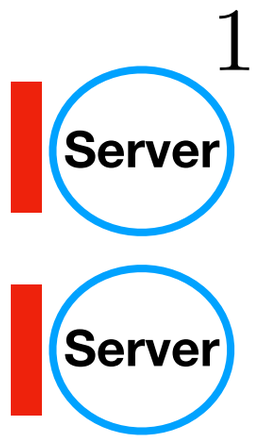
SRPT- short jobs remain in system for short time

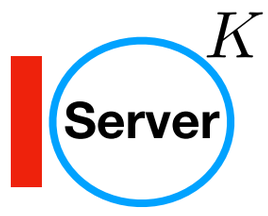
LCFS- w/o job size information, long jobs remain in system for long

Technical - construction of potential (Lyapunov) function is easy

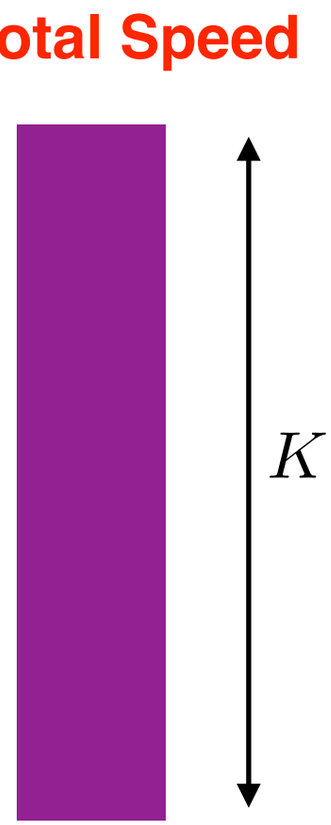
Intuition LCFS-EQUI

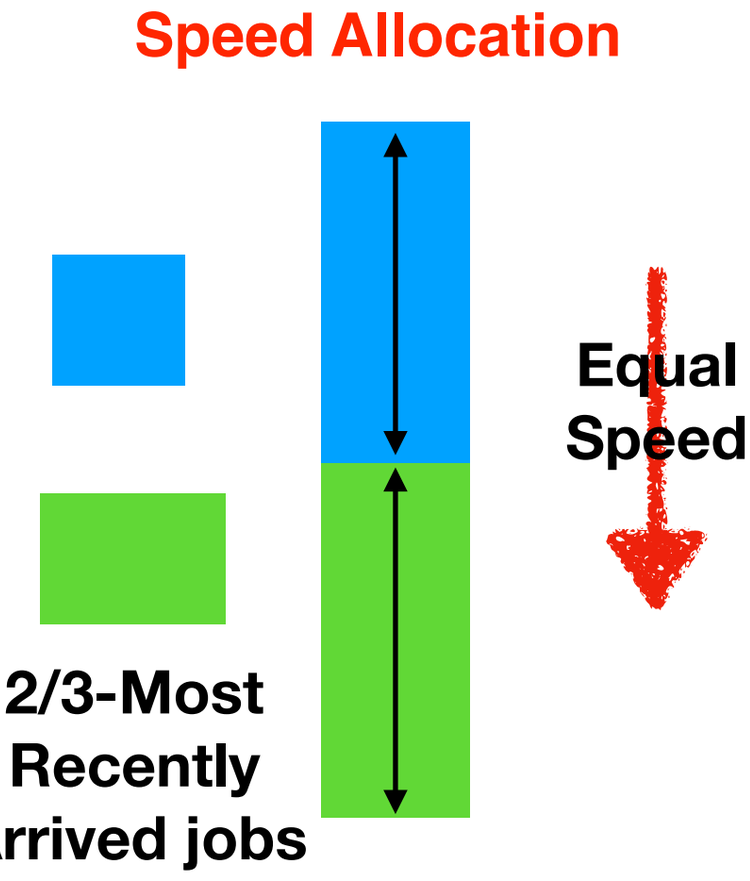






Multiple Servers





SRPT- short jobs remain in system for short time

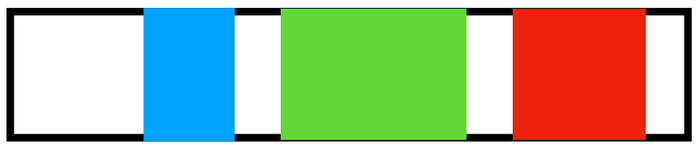
LCFS- w/o job size information, long jobs remain in system for long

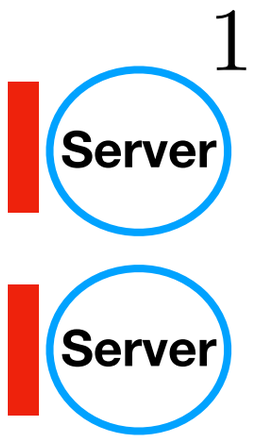
Technical - construction of potential (Lyapunov) function is easy

EQUI : with equal speed, analysis is easy

Guarantee LCFS-EQUI

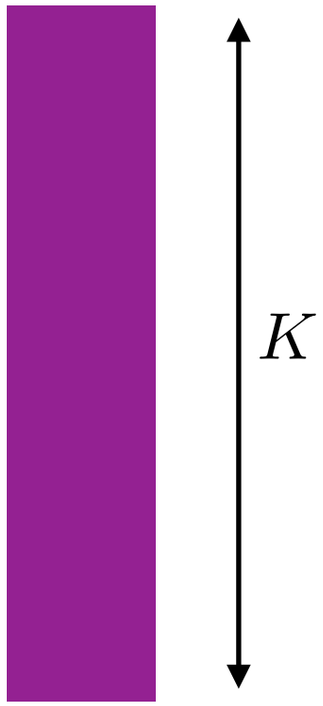
packets arrivals



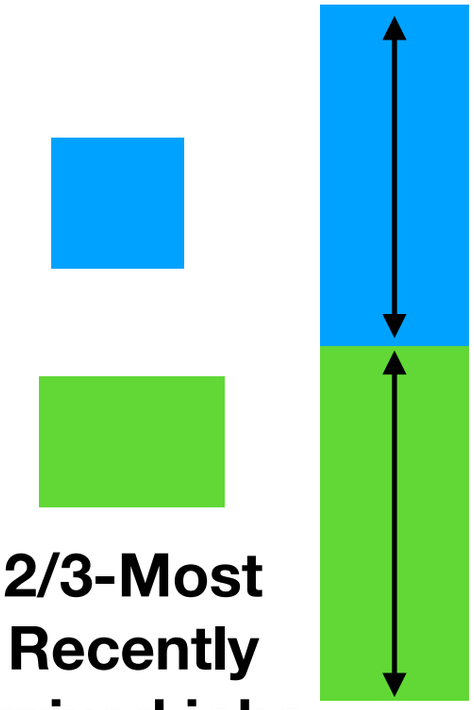


Multiple Servers

Total Speed



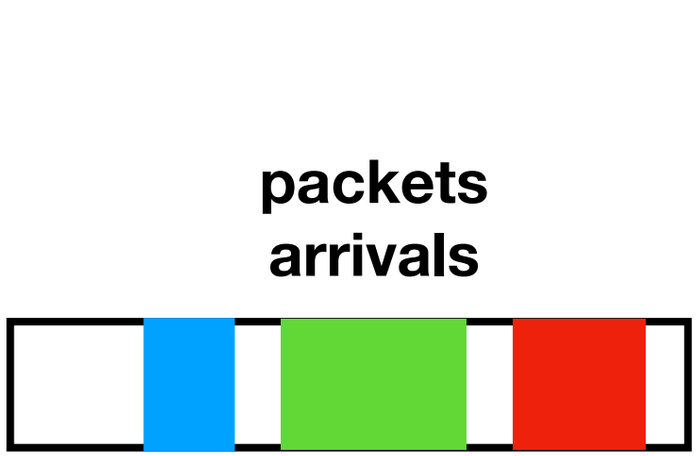
Speed Allocation

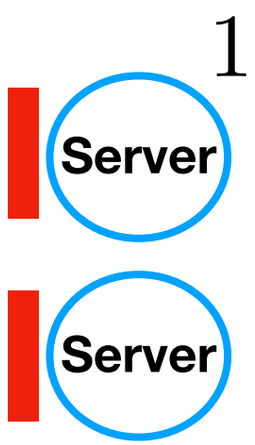


2/3-Most Recently Arrived jobs

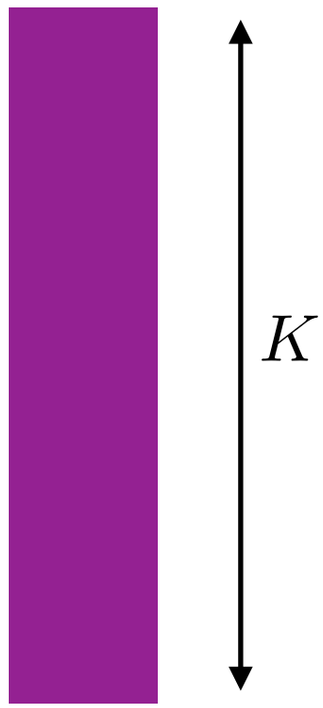
Equal Speed

Guarantee LCFS-EQUI

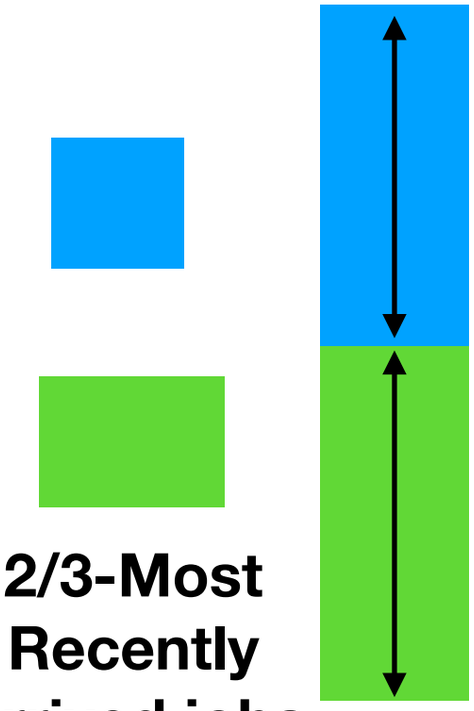




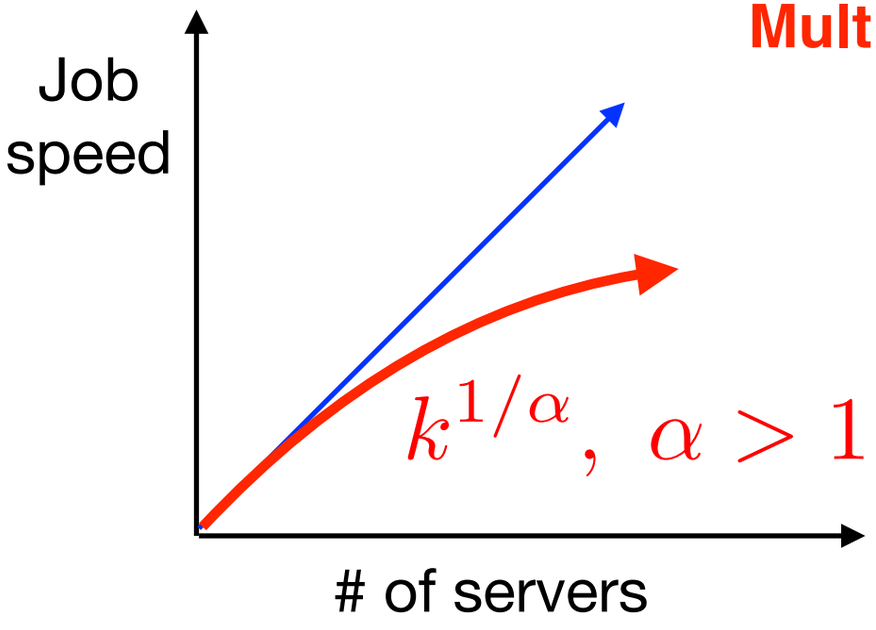
Total Speed



Speed Allocation

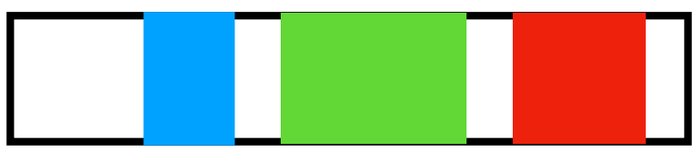


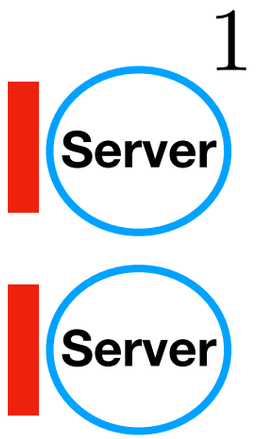
Multiple Servers



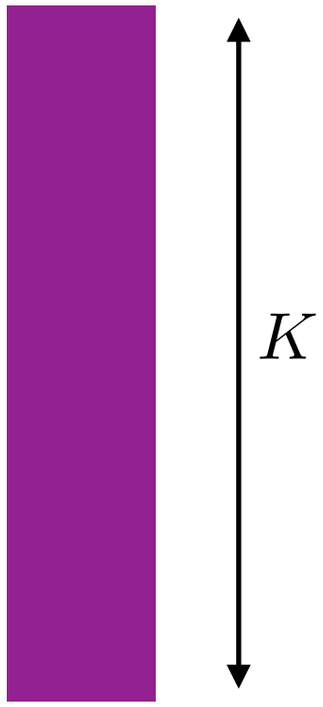
Guarantee LCFS-EQUI

packets arrivals

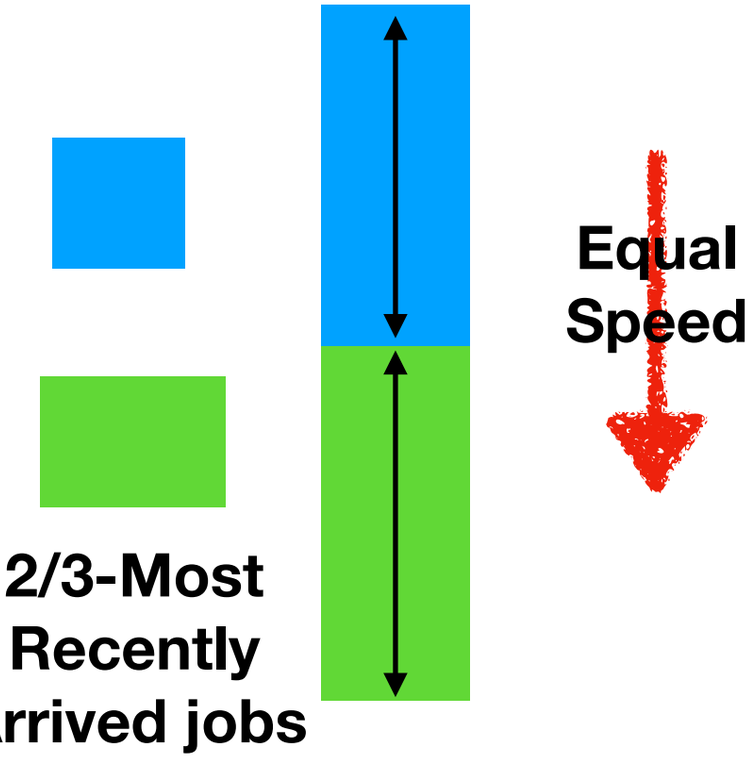




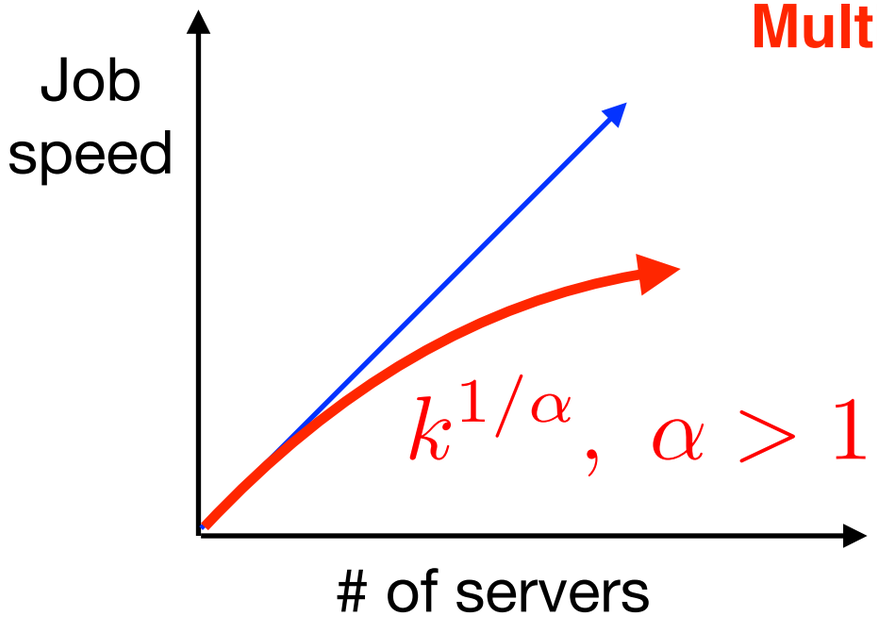
Total Speed



Speed Allocation



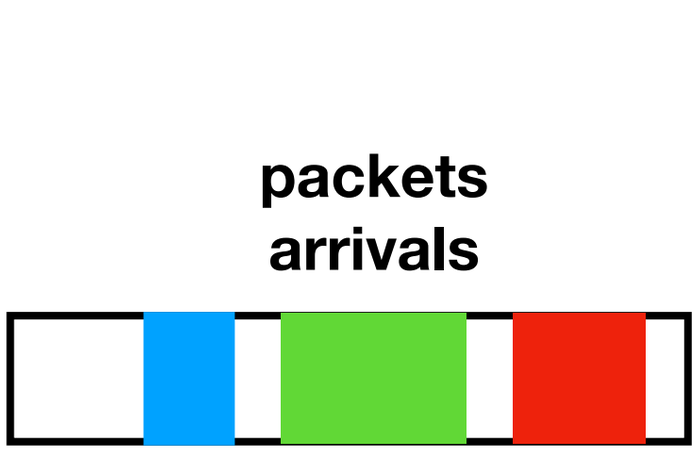
Multiple Servers

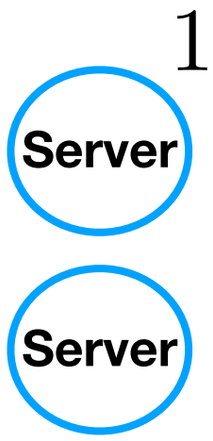


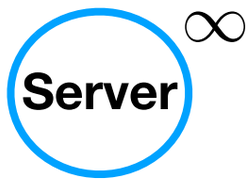
For a suitable choice of fraction β

Competitive ratio is a constant that only depends on exponent α

General Problem

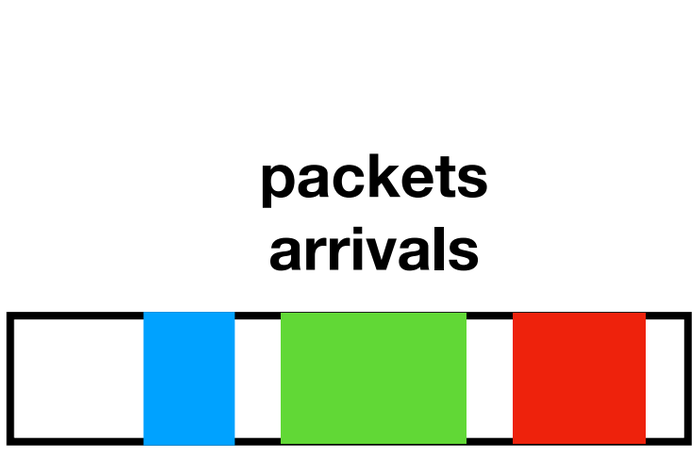


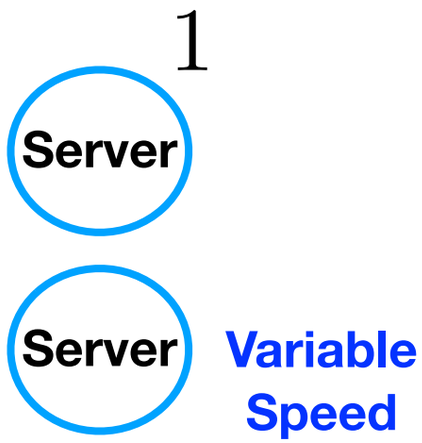


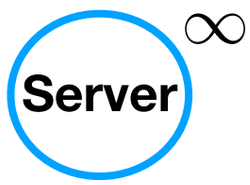


Unlimited # of Servers

General Problem

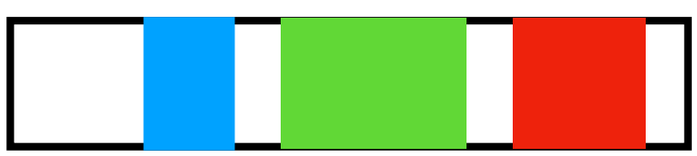


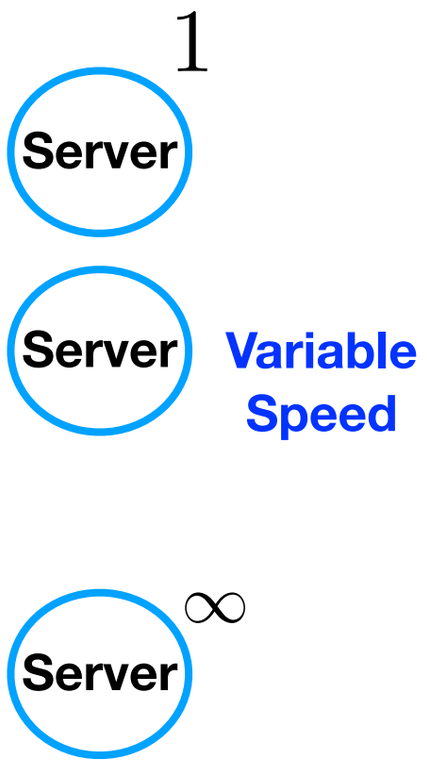




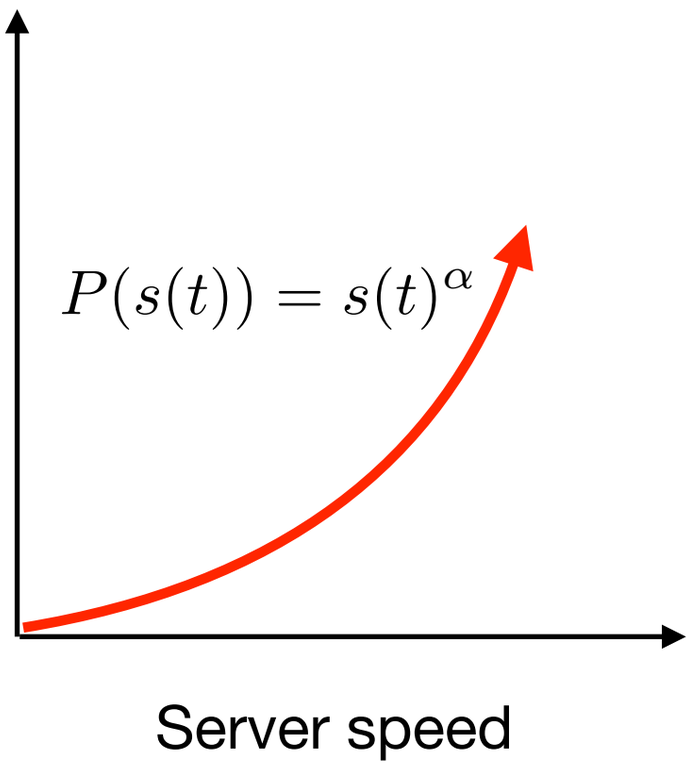
Unlimited # of Servers

General Problem

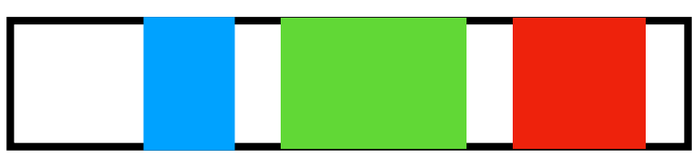


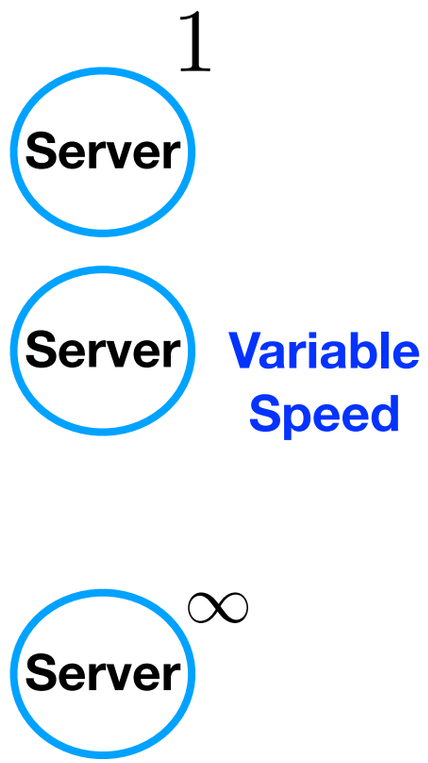


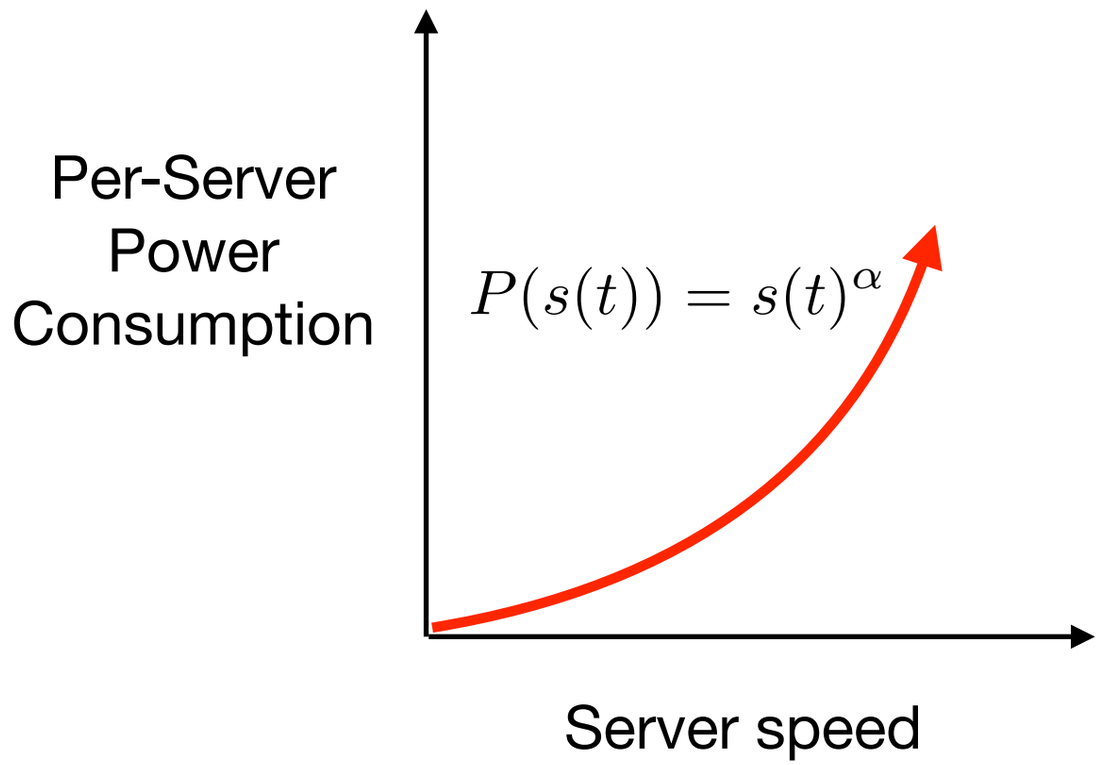
Unlimited # of Servers



General Problem







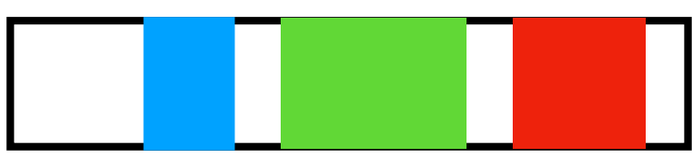
Unlimited # of Servers

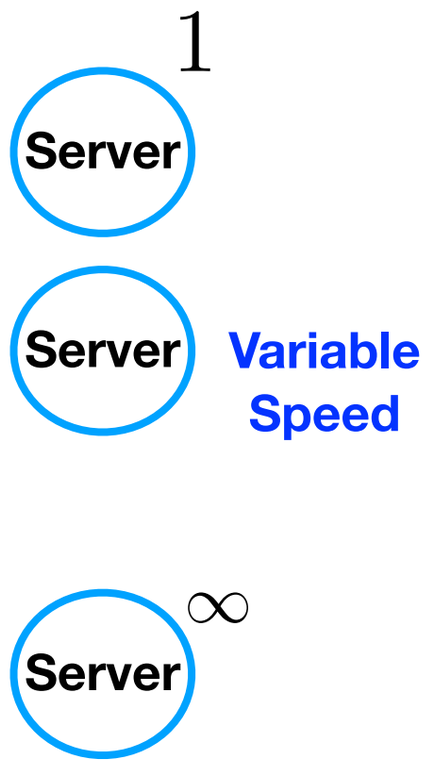
Obj: ***min*** **total flow time**

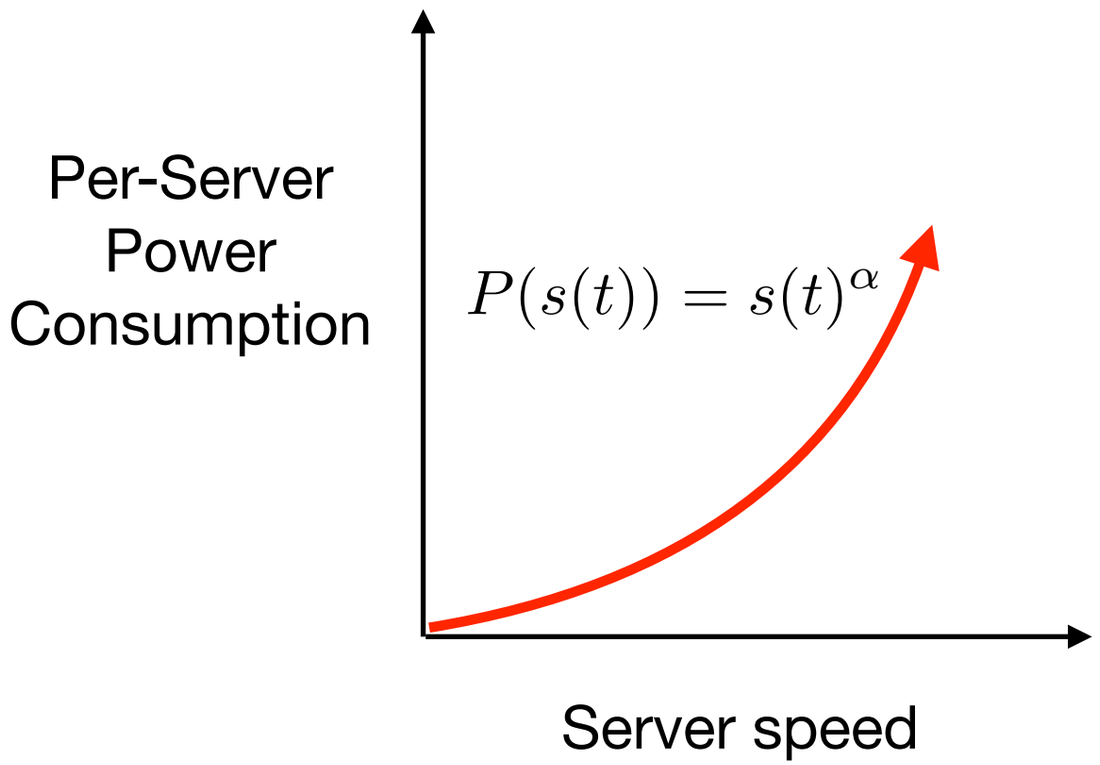
$$\int n(t)dt$$

n(t) **number of outstanding jobs at time t**

General Problem







Unlimited # of Servers

Obj: ***min*** **total flow time**

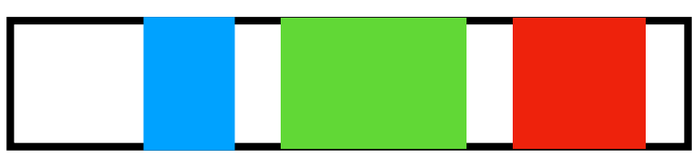
$$\int n(t)dt$$

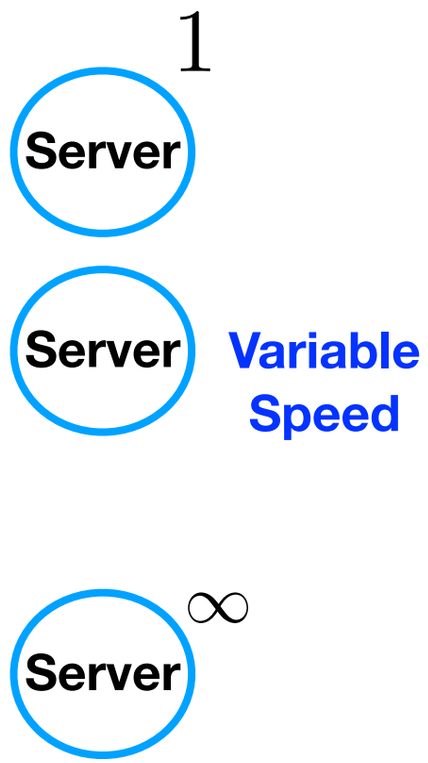
n(t) **number of outstanding jobs at time t**

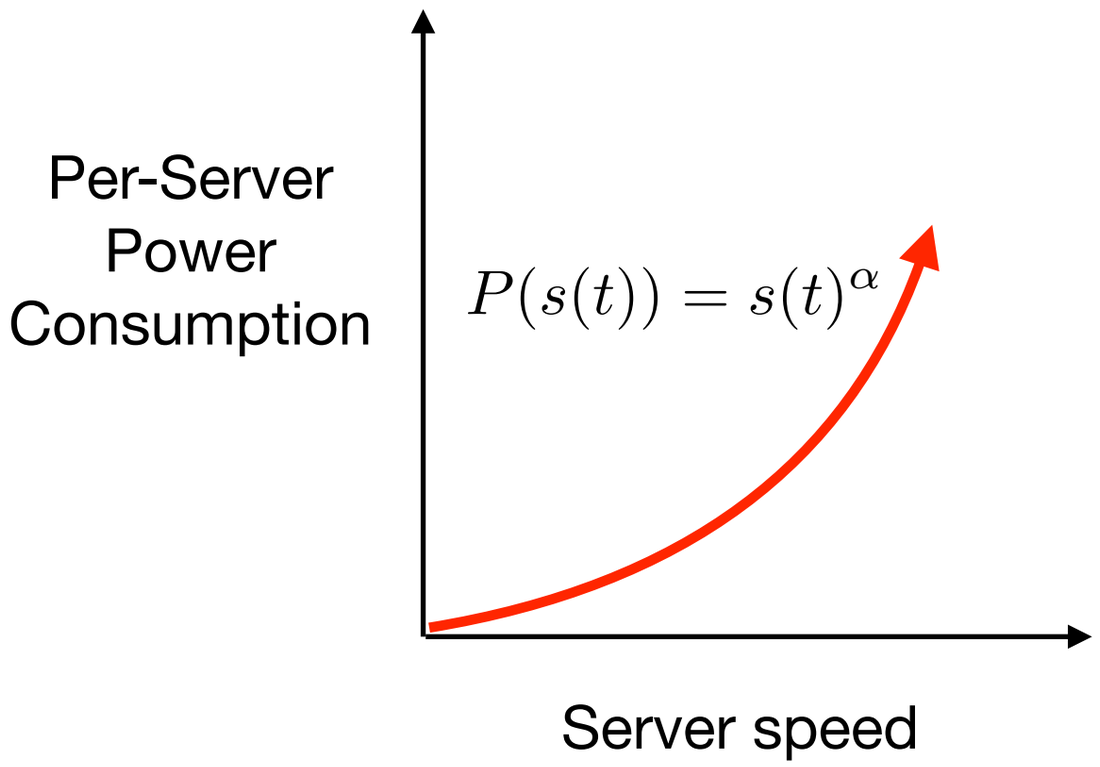
subj: sum-power constraint across all servers

$\mathcal{P}$

General Problem







Unlimited # of Servers

Obj: ***min*** **total flow time**

$$\int n(t)dt$$

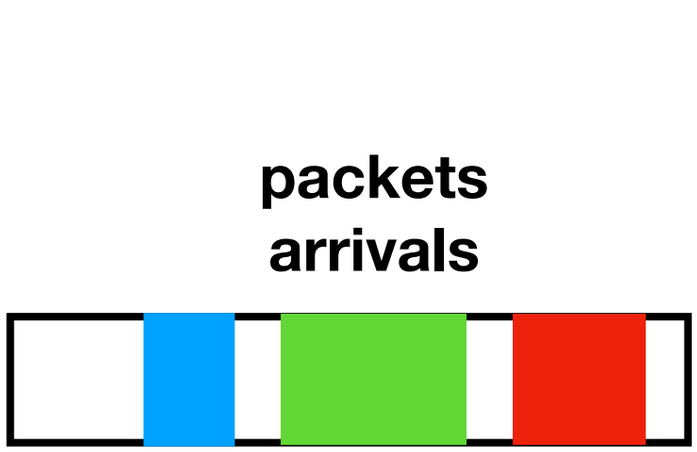
n(t) **number of outstanding jobs at time t**

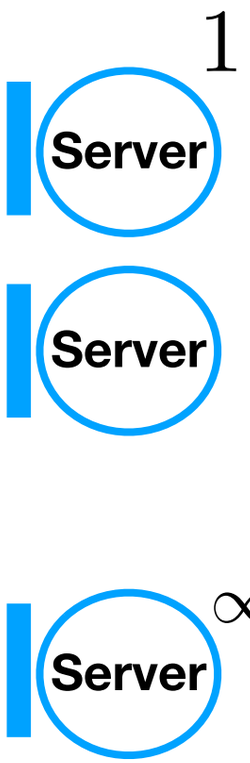
subj: sum-power constraint across all servers

$\mathcal{P}$

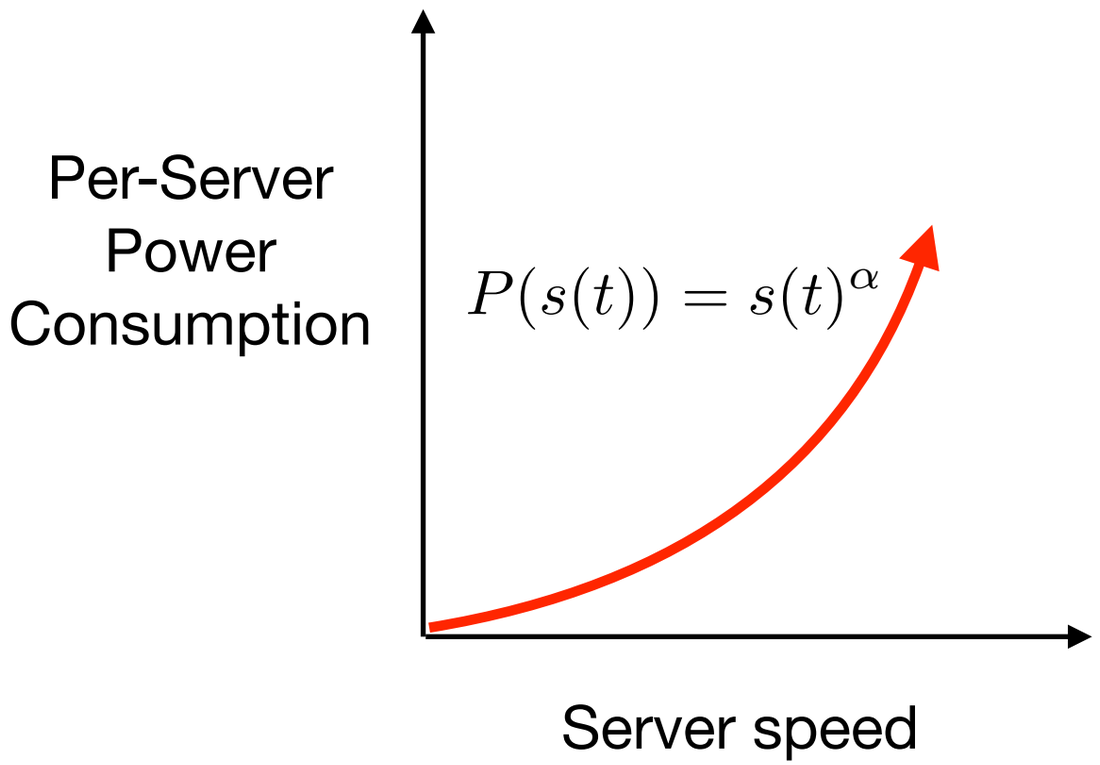
Decision : speed of each job/server

Result - General Problem

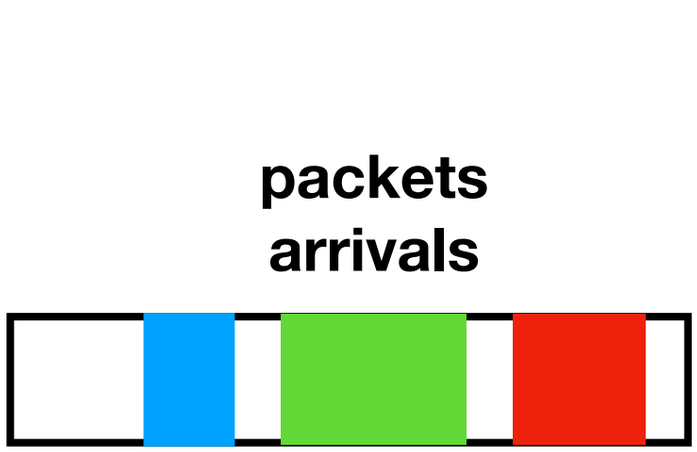


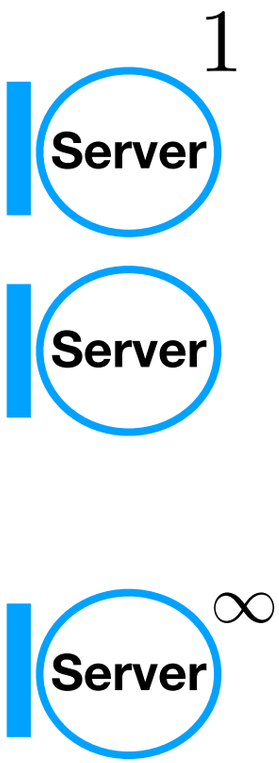


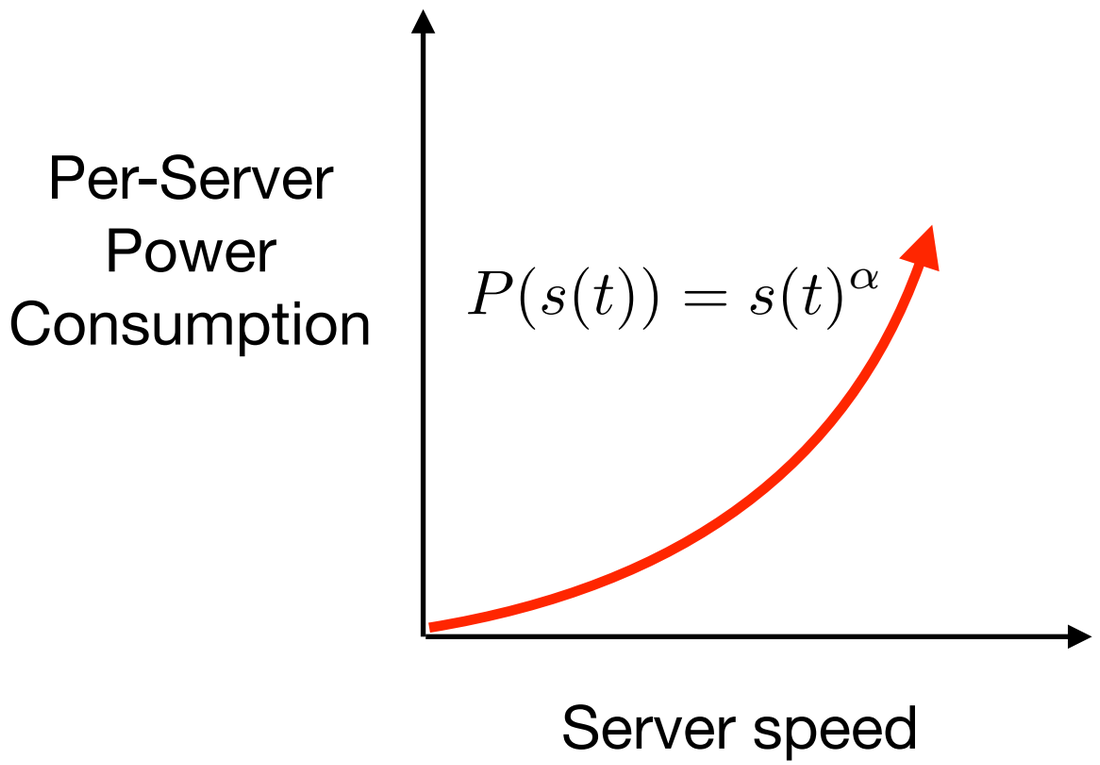
Unlimited # of Servers



Result - General Problem







Unlimited # of Servers

Competitive ratio is a constant that only depends on exponent α

Conclusions

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An online algorithm for an important job-scheduling problem

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Fractional LCFS+Equal Speed algorithm is constant competitive

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Open Questions

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Competitive ratio of heSRPT algorithm (that is locally optimal) ?

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Open Questions

Competitive ratio of heSRPT algorithm (that is locally optimal) ?

Lower Bound on the competitive ratio of any online algorithm?