Coalition Formation Resource Sharing Games in Networks

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ABSTRACT

Cooperative game theory deals with systems where players want to cooperate to improve their payoffs. But players may choose coalitions in a non-cooperative manner, leading to a coalition-formation game. We consider such a game with several players (willing to cooperate) and a possible adamant player (unwilling to cooperate) involved in resource-sharing. Here, the strategy of a player is the set of players with whom it wants to form a coalition. Given a strategy profile, an appropriate partition of coalitions is formed; players in each coalition maximize their collective utilities leading to a non-cooperative resource-sharing game among the coalitions, the (unique) utilities at the resulting equilibrium are shared via Shapley-value; these shares define the utilities of players for the given strategy profile in the coalition-formation game. We also consider the utilitarian solution to derive the price of anarchy.

KEYWORDS

Resource sharing game, Kelly's mechanism, Coalition formation, Partition form game, Utilitarian solution and Price of Anarchy

ACM Reference Format:
Shiksha Singhal and Veeraruna Kavitha. 2021. Coalition Formation Resource Sharing Games in Networks. In *Proceedings of* . ACM, New York, NY, USA, 2 pages. https://doi.org/10.1145/nnnnnnn.nnnnnnn

1 INTRODUCTION

Resource sharing problem is a well-known problem that aims to find an optimal allocation of shared resources. Wireless networks existing in the same region compete to obtain larger spectrum shares to cater for ever-growing traffic demands. It is well known that online auctions [9] can be used to achieve optimal resource allocations. These auctions majorly use a 'proportional allocation algorithm' (Kelly's mechanism [12]) which is also considered in a variety of other contexts; e.g., [13] considers real-time performance in time-shared operating systems, [6] considers rate allocation for communication networks, [8] considers resource allocation in wireless network slicing, etc. In this mechanism ([12]), the resource allocated to any player is proportional to its bid and inversely proportional to the weighted sum of bids of all players, with the weights representing the influence factors.

We also consider Kelly's mechanism, but, with very important differentiating features: i) possibility of cooperation among the willing players; and, ii) the possible presence of an adamant player, not interested in cooperation. For example, most of the literature related to spectrum auctions utilising Kelly's mechanism, considers non-cooperative players [3, 9]. As mentioned in [9], it would be interesting to see if the agents can buy spectrum together and divide the allocated spectrum amicably. However, one cannot rule out the existence of players who are not interested in this kind of collaborations, and these form the *adamant player* of our paper. Majority of the analysis related to cooperative games discuss the emergence of grand coalition as a successful partition but one can find many example scenarios, in which a partition of strict coalitions (subsets) of N might emerge at some appropriate equilibrium. Thus in this paper, we consider a relevant aspect for investigation: *when and which subset of willing players find it beneficial to collaborate.* We consider such a study using 'non-cooperative coalition formation games'.

Many applications can be modelled using this framework, e.g., spectrum allocation [9], cloud computing [14], network slicing [8], allocation of advertisement space [7, 11], market share [10], etc.

2 PROBLEM DESCRIPTION

Consider a system with $(n+1)$ players involved in a resource sharing game (RSG). Let $N = \{0, N_C\},\ N_C := \{1, 2, \cdots, n\}$ denote the set of players (willing to cooperate) along with an adamant player indexed by 0. The n players in N_C are willing to cooperate with each other if they can obtain higher individual share while the adamant player is not interested in cooperation. The utility of players is proportional to their actions which also includes a proportional cost. Thus, when players choose respective actions $(a_0, a_1, \cdots, a_n)$, the utility of player i equals

$$\varphi_i = \frac{\lambda_i a_i}{\sum_{j=0}^{n} \lambda_j a_j} - \gamma a_i \ \forall\ i \ \in \ N, \tag{1}$$

where γ represents the cost factor, λ_i represents the influence factor of i^{th} player and a_i represents the action of i^{th} player.

When the players are looking for opportunities to form coalitions and work together; each player proposes a strategy which is the subset of players with whom it wants to form a coalition [1]; thus, $x_i \subseteq N_C$ is a strategy of player i. Given a strategy profile $\underline{\mathbf{x}}$ of all players in N_C, set/collection(s) of coalitions emerges (which satisfies certain rules, given in [4]); say $\mathcal{P}(\underline{\mathbf{x}}) = \{S_0, S_1, \cdots, S_k\}$ represents the partition of N into different coalitions where $S_0=\{0\}$ denotes the adamant player; further, multiple partitions can emerge from a strategy profile. The players in *coalition* S_i choose their actions together and hence the utility of a coalition is given by:

$$\varphi_{S_m}(\mathbf{a}_m, \mathbf{a}_{-m}) = \frac{\sum_{l \in S_m} \lambda_l a_l}{\sum_{l=0}^{n} \lambda_l a_l} - \gamma \sum_{l \in S_m} a_l;\ m \geq 0 \tag{2}$$

$$\text{where, } \mathbf{a}_m = \{a_i, i \in S_m\}, \mathbf{a}_{-m} = \{a_i, i \notin S_m\}, \forall S_m \in \mathcal{P},$$

which is the sum of their individual utilities. The players will now try to derive maximum utility for their own coalition and hence

ACM ISBN 978-x-xxxx-xxxx-x/YY/MM…$15.00
https://doi.org/10.1145/nnnnnnn.nnnnnnn

there would again be a non-cooperative game, but now among coalitions. Thus we have a reduced RSG (for every $\mathcal{P}$) with each coalition representing one (aggregate) player and the utilities given by (2); utility of any coalition equals that at the corresponding NE.

Theorem 2.1. *The game with utilities $\{\varphi_{S_m}\}$ as in (2), can have multiple NE, but the utilities at NE are unique. There exists a $M^{\mathcal{P}} \le k$ such that only the coalitions in $\mathcal{J}^* = \{S_1, \cdots, S_{M^{\mathcal{P}}}\}$ get non-zero utilities (with $\bar{\lambda}_m^{\mathcal{P}} = \max_{i\in S_m} \lambda_i$ arranged in decreasing order). The unique NE-utility for any $m \le k$ is given by ,*

$$\varphi^*_{S_m}(\mathcal{P}) = \left(\frac{s^{\mathcal{P}} - \frac{M^{\mathcal{P}}-1}{\bar{\lambda}_m^{\mathcal{P}}}}{s^{\mathcal{P}}} \right)^2 \mathbb{1}_{S_m \in \mathcal{J}^*}, \text{ with,} \tag{3}$$

$$M^{\mathcal{P}} := \max\left\{ m \le k : \sum_{i=1}^{m} \frac{1}{\bar{\lambda}_i^{\mathcal{P}}} - \frac{m-1}{\bar{\lambda}_m^{\mathcal{P}}} > 0 \right\}, \text{ and, } s^{\mathcal{P}} = \sum_{m=1}^{M^{\mathcal{P}}} \frac{1}{\bar{\lambda}_m^{\mathcal{P}}}. \quad \blacksquare$$

This utility is divided among the members of the coalition using the well-known *Shapley value* (computed within the coalition) as in [5], *which simplifies to equal shares for symmetric players.* For the case when a strategy profile $\underline{\mathbf{x}}$ leads to multiple partitions, the utility $U_i(\underline{\mathbf{x}})$ of a player is defined to be the minimum utility among all possible partitions, similar to α-effectiveness in [2].

Thus, we have a non-cooperative strategic form game. Opposed to the usual practice of studying the equilibrium strategy profile, here we are more interested in the equilibrium partitions and hence in line with the concept of Nash Equilibrium (NE), we define a solution concept called NE-partitions. We say any partition $\mathcal{P}$ that emerges from a NE strategy profile, is a *NE-partition.*

However, there is a possibility that a NE strategy profile can lead to multiple NE-partitions; this has an inherent instability associated with it. Thus, we propose another solution concept called *U-stable partitions.* We say a partition $\mathcal{P}$ to be U-stable if the corresponding natural strategy profile, i.e., $x_i = S_j$ for all $i \in S_j \in \mathcal{P}$ and for all j, is a NE. We also consider solutions that optimize social objective function to derive the Price of Anarchy, P_{oA}. The details can be found in [4].

This is the problem setting and our aim is to study the coalitions/partitions that emerge out successfully (at an appropriate equilibrium), when the players seek opportunities to come together in a non-cooperative manner.

3 RESULTS

We consider a coalition formation game with players exploring cooperation opportunities in a non-cooperative manner, where the utilities of players/coalitions are resultant of a resource sharing game among coalitions. Following are the important results:

(1) With equal or almost equal (influence) players, no one collaborates at equilibrium (if $n > 4$) and coarser partitions (some players collaborate) emerge at NE for smaller n; and the former case does not depend upon adamant player, while latter case depends. For large n (i.e., $n > 4$), refer to Corollary 2 and for smaller n, see Section 5.7 of [4].
(2) In all cases, the P_{oA} increases with n (as $O(n)$) and with increase/decrease in the strength of adamant player (see Tables 4-6 and 8 in [4]).
(3) Interestingly, none of the partitions are coalitionally stable for the case with $n > 4$ and equal (influence) players (see [4, Theorem 12]).
(4) Surprisingly, when the players are significantly different, every partition is stable against unilateral deviations (see [4, Theorem 5]).
(5) Grand coalition is the only partition stable against coalitional deviations (for a special case) with highly asymmetric players (see [4, Theorem 11]).
(6) Interestingly, grand coalition is the only utilitarian partition which is also stable against coalitional deviations (for the same special case mentioned above).
(7) For the system with intermediate players, the number of U-stable partitions (stable against unilateral deviations) increase as asymmetry (a measure of differences in the influence factors of various players) increases (see Section 8.4 for numerical results in [4]).
(8) Lastly and more interestingly, it is the highest and the lowest capacity players that first find it beneficial to collaborate (see Theorems 7-9 for theoretical results and Section 8.4 for numerical results in [4]).

4 ACKNOWLEDGMENTS

The work of the first author is partially supported by Prime Minister's Research fellowship (PMRF), India.

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