Learning traffic correlations in multi-class queueing systems by sampling workloads

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Joint work with *Michel Mandjes* (U. of Amsterdam)

November 10th, 2021
IFIP Performance

Motivation: Backbone of the Internet

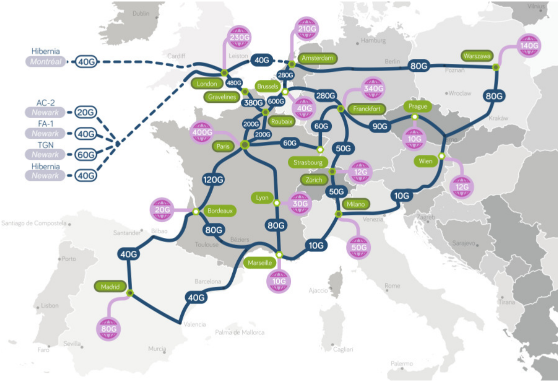

Figure: OVH Europe network

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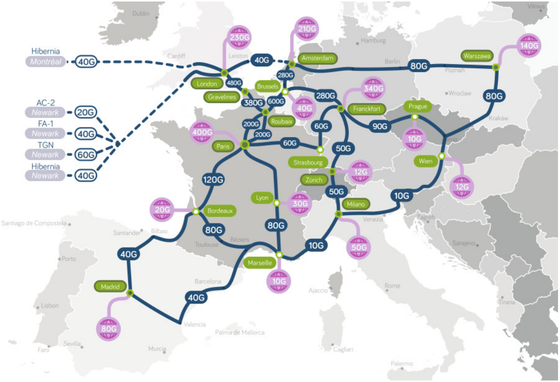

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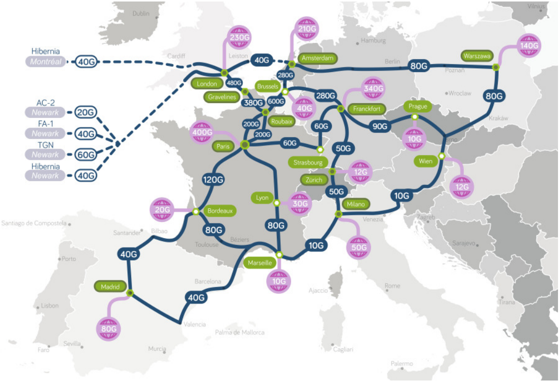

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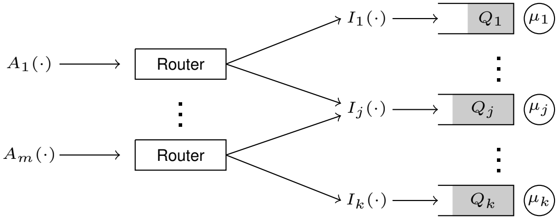
Features:

- **Traffic:** Highly aggregated
- **Routing:** Mostly static

Stylized model: Static load balancing



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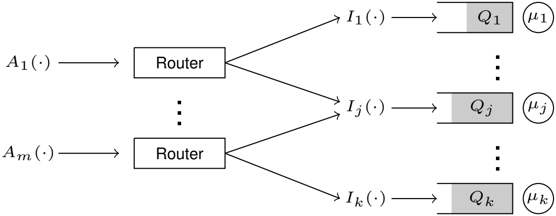

Assumptions:

- **Arrivals:** $A(\cdot)$ is Gaussian with known rate $\lambda \in \mathbb{R}_+^m$ and unknown covariance matrix $\Sigma : \mathbb{R}^2 \to \mathbb{R}^{m \times m}$

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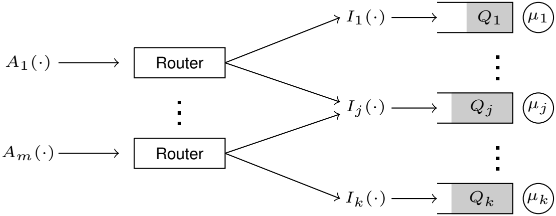

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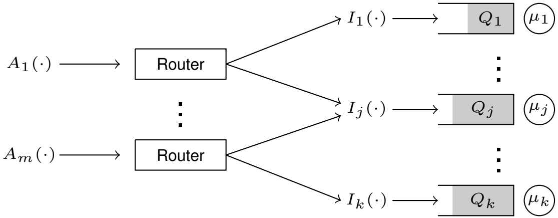

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Objective: Learn

$$R^* \in \arg\min \left\{ \max_{i \in \{1,\dots,k\}} \left\{ \mathbb{P}\big(Q_i(R) > b_i\big) \right\} \right\}$$

Additional assumptions on $A(\cdot)$

Assumption: Many-sources regime

$$A(\cdot) = A^{(n)} = \frac{1}{n}\sum_{i=1}^{n} X^{(i)}(\cdot),$$

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For simplicity: multivariate fractional Brownian motions (mfBm)

$$Cov\left(A_i^{(n)}(t),\ A_j^{(n)}(s)\right) = \frac{\sigma_i \sigma_j \rho_{i,j}}{2n}\left(|t|^{2H} + |s|^{2H} - |s-t|^{2H}\right)$$

- Hurst parameter: $H \in (0,1)$
- Variance: $\sigma_i^2 > 0$
- Correlation: $\rho_{i,j} \in [-1,\ 1]$

How do we find the optimal routing matrix?

Optimization with indirect learning

Algorithm:

- Start with initial routing matrix R_0

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Advantages:

- Queue lengths are easier to estimate than covariances
- Fast convergence

First inversion procedure

From marginal queue lengths to variances

[Mandjes & van de Meent (2009) Resource dimensioning through buffer sampling]

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$$Q_i^{(n)} \xrightarrow{\text{“}Inversion\text{”}} Var\left(I_i^{(n)}(\cdot)\right)$$

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based on the large-deviations principle

$$\lim_{n\to\infty} -\frac{1}{n}\log\left(\mathbb{P}\left(Q_i^{(n)} > b\right)\right) = \inf_{t<0}\left\{\frac{\left[b-(\mu_i-\overline{\lambda}_i)t\right]^2}{2nVar\left(I_i^{(n)}(t)\right)}\right\}$$

From marginal queue lengths to variances

Variance estimator:

$$\hat{V}_{i,\epsilon}^{(n,N)}(t) \triangleq \inf_{b\in[\epsilon,1/\epsilon]} \left\{ \frac{\left[b-(\mu_i-\overline{\lambda}_i)t\right]^2}{-2\log\left(\hat{\mathbb{P}}_N\left(Q_i^{(n)}>b\right)\right)} \right\} \overset{?}{\approx} Var\left(I_i^{(n)}(t)\right)$$

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Theorem

Fix $t<0$. We have

$$\lim_{n\to\infty}\lim_{N\to\infty} n\left|\hat{V}_{i,\epsilon}^{(n,N)}(t)-Var\left(I_i^{(n)}(t)\right)\right|=0, \quad a.s.,$$

for all ϵ small enough.

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- For each of the k queues we obtain a linear equation

$$\hat{V}_{i,\epsilon}^{(n,N)}(\cdot) \approx Var\left(I_j^{(n)}(\cdot)\right) = \sum_{j=1}^{m}\sum_{q=1}^{m} R_{j,i}R_{q,i}Cov\left(A_j^{(n)}(\cdot),\; A_q^{(n)}(\cdot)\right)$$

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Use joint queue lengths to get more equations directly?

Second inversion procedure

From pair-wise joint queue lengths to covariances

Work flow:

$$A^{(n)}(\cdot) \xrightarrow{Routing} \left(I_i^{(n)}(\cdot),\, I_j^{(n)}(\cdot)\right) \xrightarrow{Queueing} \left(Q_i^{(n)},\, Q_j^{(n)}\right)$$

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Feasibility of the inversion

Covariance estimator:

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Theorem

If $I^{(n)}(\cdot)$ is short-range dependent ($H \leq 1/2$), and $I_i^{(n)}(\cdot)$ and $I_j^{(n)}(\cdot)$ are non-negatively correlated ($\rho_{i,j} \geq 0$), then

$$\lim_{n\to\infty} \lim_{N\to\infty} n \left| \hat{C}_{i,j,\epsilon}^{(n,N)}(t,t) - Cov\left(I_i^{(n)}(t),\, I_j^{(n)}(t) \right) \right| = 0, \quad a.s.,$$

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- If $m \leq k$, these are enough for a single R

Fundamental limitation

Theorem

If $I^{(n)}(\cdot)$ is long-range dependent ($H > 1/2$), and $I_i^{(n)}(\cdot)$ and $I_j^{(n)}(\cdot)$ are negatively correlated ($\rho_{i,j} < 0$), then $Cov\left(I_i^{(n)}(t),\, I_j^{(n)}(t)\right)$ **cannot be recovered** from the large deviations behavior of $c_i Q_i^{(n)} + c_j Q_j^{(n)}$.

There is an inherent loss of information!

Why do we have this limitation?

Intuition with queues as reflected processes

Note: $Q_i^{(n)}(\cdot)$ and $Q_j^{(n)}(\cdot)$ are the reflection at 0 of

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- Magnitude of correlation is lost in the reflection at 0

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  $\Rightarrow$ Inputs' covariances **cannot** be recovered
- Can be extended to multi-path routing in acyclic networks (needs much more involved large-deviations results)

Thank you!