

# Competitive Bidding Strategies for Online Linear Optimization with Inventory Management Constraints

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## ABSTRACT

This paper develops competitive bidding strategies for an online linear optimization problem with inventory management constraints in both cost minimization and profit maximization settings. In the minimization problem, a decision maker should satisfy its time-varying demand by either purchasing units of an asset from the market or producing them from a local inventory with limited capacity. In the maximization problem, a decision maker has a time-varying supply of an asset that may be sold to the market or stored in the inventory to be sold later. In both settings, the market price is unknown in each timeslot and the decision maker can submit a finite number of bids to buy/sell the asset. Once all bids have been submitted, the market price clears and the amount bought/sold is determined based on the clearing price and submitted bids. From this setup, the decision maker must minimize/maximize their cost/profit in the market, while also devising a bidding strategy in the face of an unknown clearing price. We propose **DEMBID** and **SUPBID**, two competitive bidding strategies for these online linear optimization problems with inventory management constraints for the minimization and maximization setting respectively. We then analyze the competitive ratios of the proposed algorithms and show that the performance of our algorithms approaches the best possible competitive ratio as the maximum number of bids increases. As a case study, we use energy data traces from Akamai data centers, renewable outputs from NREL, and energy prices from NYISO to show the effectiveness of our bidding strategies in the context of energy storage management for a large energy customer participating in a real-time electricity market.

## Keywords

Online bidding strategy design, inventory management, energy storage systems, electricity market

## 1. INTRODUCTION

In this paper, we study online linear optimization problems with inventory management constraints in a bidding

scenario. In each time slot, a decision maker must satisfy an online arrival of asset demand  $d(t)$  in the cost minimization setting, while it receives an online supply of asset  $u(t)$  in the profit maximization setting. The asset may be bought at a cost or sold at a profit with the online arrival of price  $p(t)$ , where the buying amount  $x(t)$  or selling amount  $y(t)$  must also be decided online. With inventory management, the decision maker may utilize an inventory of capacity  $B$  to store the asset between time slots. With bidding strategies, the price  $p(t)$  and decision variables  $x(t)$ ,  $y(t)$  are not immediately known in the current time slot - the decision maker must first submit a set of bids on the asset, after which  $p(t)$ ,  $x(t)$ , and  $y(t)$  are made known based on the submitted bids. The problems are considered **PROFITMAX** and **COSTMIN** for the maximization and minimization versions.

Online linear optimization is well-studied and recent work on the inventory management variant has also provided worst-case optimality guarantees for both maximization and minimization versions [6, 4]. The bidding setting, however, introduces new challenges in online algorithm design because the online price inputs are not even known in the current time slot. Specifically, in most prior work in online algorithms, even though the future input is assumed to be unknown, the online input for the current slot is known accurately. In this paper, we consider a case that the online input for the current slot is also unknown. In such scenario, the online decision maker participates in a market-based bidding mechanism to determine the amount of asset that should be traded (sold or bought) in the market.

From the technical perspective, the unknown price exacerbates the challenges of online algorithm design such that prior algorithms that are designed for the settings with known current price are not applicable for the bidding scenarios. Specifically, prior online algorithms are developed based on carefully-designed threshold functions that determine the trading amount based on the current utilization of the inventory and the known current price [6, 4]. In the bidding scenario with unknown price, it is not possible to directly apply those threshold functions anymore and hence it is needed to design strategies for both variants of the problem, i.e., **PROFITMAX** and **COSTMIN**.

In addition to introducing new technical challenges, both **PROFITMAX** and **COSTMIN** are of significant practical rel-

evance for the timely problem of bidding strategy design for participation of energy storage systems in real-time electricity markets. The bidding strategy for storage participation in markets is different from traditional energy resources since energy storage systems are flexible resources that pose unique physical and operational characteristics. The first unique challenge is the *flexibility in the generation*: energy storage systems are flexible in releasing energy, i.e., it is possible to store energy and release it at future times if that is more beneficial. Traditionally, the trade in the electricity market was based on the fact that the energy could not be stored, and the market operations and bidding strategies were designed based on this fundamental assumption.

The second challenge is *uncertainty*: energy storage systems often provide energy alongside variable renewable generation, which results in inherent uncertainty in their energy generation. In addition, the energy market is highly uncertain and market prices change dynamically based on supply and demand fluctuations.

The above two challenges make bidding strategy design for storage-assisted parties a fundamentally different problem than the traditional energy market participants. However, the problem of interest in this paper can fully capture both challenges: the flexibility in generation could be captured by the inventory management constraints, and the uncertainty could be captured by the online algorithm design framework.

## 1.1 Our Contributions

In this paper, we tackle the bidding strategy design problems for COSTMIN and PROFITMAX using a principled approach grounded on online algorithm design [1]. This framework enables designing bidding strategies that are provably robust against uncertainty. This paper makes the following contributions:

### 1.1.1 Algorithm Design

We develop two online algorithms, DEMBID for COSTMIN and SUPBID for PROFITMAX, and analyze their performance using *competitive ratio* as a well-established metric for online algorithms. The competitive ratio is defined as maximum ratio between an *offline* optimal algorithm with full information on inputs and the limited information online algorithm. The design of our bidding strategies is inspired by prior storage management algorithms in a simplified setting in which the market price is known in advance [5, 6, 4]. However, in bidding strategy design problems, the decision maker submits bids without knowing the market price. This may result in declining the bid, jeopardizing the feasibility of the online solution. Furthermore, the actual amount of asset traded will also be uncertain, which introduces the challenge of underbidding and overbidding on the asset due to the unknown price. Hence, the existing algorithms [5, 6, 4] are not applicable to the bidding scenario. We utilize the possibility of submitting multiple bids [3] to the market and resolve these challenges for bidding strategies for both demand and supply sides.

### 1.1.2 Competitive analysis.

As the theoretical contribution, we characterize the competitive ratio of DEMBID and SUPBID as a function of number of bids, and show the competitive ratios approach those values of the basic algorithms [6, 4] as the number of bids grows.

**THEOREM 1.** *DEMBID achieves the competitive ratio of  $\alpha_{DEMBID}$  as*

$$\alpha_{DEMBID} = \alpha \cdot \left(\frac{\theta}{\alpha}\right)^{1/(m-1)}$$

for  $m > 1$ , where  $m$  is the maximum number of bids, and  $\alpha$  is the competitive ratio of the basic algorithm in [4]

**THEOREM 2.** *SUPBID achieves the competitive ratio of  $\alpha_{SUPBID}$  as*

$$\alpha_{SUPBID} = (\ln \theta + 1) \cdot \theta^{1/(m-1)}$$

for  $m > 1$ , where  $m$  is the maximum number of bids, and  $\ln \theta + 1$  is the competitive ratio of the basic algorithm in [6].

For both algorithms as  $m \rightarrow \infty$ , the competitive ratios approach to those optimal values for the basic settings with known prices.

### 1.1.3 Empirical Evaluations

Lastly, we empirically evaluate our bidding algorithms using extensive data traces of electricity prices from NYISO, energy demands from Akamai data centers, and renewable production values from solar and wind generation. In an extensive set of experiments, the performance of our algorithms is only 5% worse than the cases in which the price of the market for the incoming slot is known in advance. In addition, our algorithms outperform alternative baseline algorithms by more than 10%, on average. Finally, our results show that as the number of available bids increases, the performance of our algorithms approach the ideal performance of algorithms which know the price in advance. We refer to [2] for the full version of our results.

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